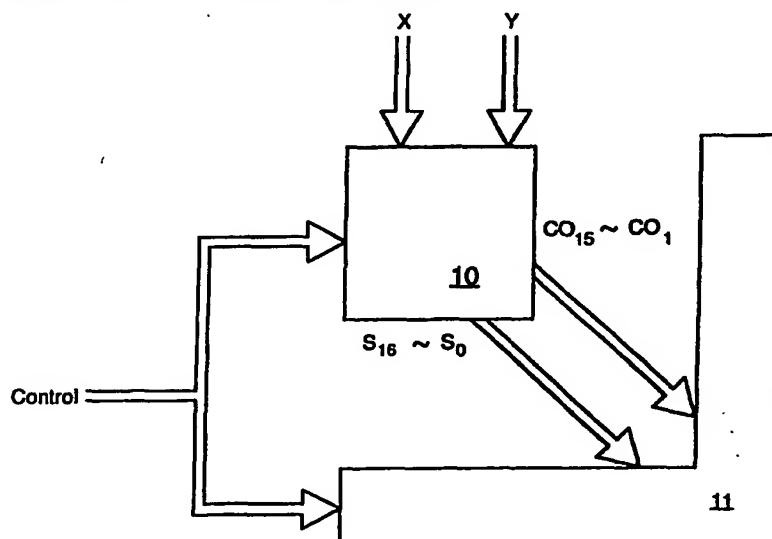


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(54) Title: UNIVERSAL MULTIPLIER-ACCUMULATOR



(57) Abstract

A high-speed circuit that performs unsigned mode, two's complement mode, and two types of mixed mode multiplication-accumulation with equal facility. The circuit is an array (10) constructed from ten different adder elements (FA1S, FA1A, FA2A, FAC, FACA, FACC, FAAC, FAAC3, HAC, and HAC2, which correspond to Figures 13-22, respectively). The array has two multiplier input operands (X and Y) and one accumulator term (Z), the three of which may be expressed as binary power expansions. Final addition of sumout and carryout terms of the array (10) is performed by final adder (11), which may comprise any of several possible adder configurations, including full carry lookahead, carry select, and conditional-sum type adders. Speed is accomplished through the use of high-speed adder elements having few gate delays, and by summing all even array rows together and all odd rows together, then adding the even sum with the odd sum in the final adder (11) using a Wallace tree technique. The circuit incorporates a high degree of regularity and interconnectivity, which facilitates compact circuit layout.

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1
UNIVERSAL MULTIPLIER-ACCUMULATOR

Field of the Invention

This invention generally relates to high-volume high-speed digital parallel processing, in particular a multiplier-accumulator integrated circuit array that will
5 accept input operands in two's complement, unsigned magnitude, and mixed modes.

Background of the Invention

A multiplier-accumulator is a computational device that multiplies two multiplier input terms and sums the
10 product with an accumulator term, providing a final output term. A parallel multiplier has the characteristic of operating on multiple bits within the terms simultaneously, that is, in parallel. High speed parallel multipliers and multiplier-accumulators and their efficient integration
15 into silicon or some other semiconductor substrate have been of interest since the early days of digital computers.

Many arithmetic circuits which are widely used in the digital arts are designed for use with two's complement binary signals. While such signals and the associated
20 circuits are particularly well adapted for performing efficiently under many circumstances, two's complement multiplication often requires the use of specialized circuits for correcting errors or characteristically incomplete results. Attempts have been made to reduce
25 specialized circuitry in two's complement arithmetic circuits. See, for example, US patent number 3,866,030 by inventors Baugh and Wooley for a two's complement parallel array multiplier.

A multiplier-accumulator that can operate equally well
30 across two's complement, unsigned, and mixed modes is

desirable. A first objective² of the present invention is, therefore, to provide a multiplier-accumulator that is universal across all modes.

5 Although a multiplier-accumulator has speed advantages when compared to equivalent circuits, more speed is always desirable. A second objective of the present invention is, therefore, to provide a multiplier-accumulator that has high operational speed.

10 It has long been recognized that an arrayed circuit structure comprised of identical or nearly-identical components facilitates circuitry layout (whether the layout is performed by hand or with computer-aided design tools). In addition, an arrayed circuit structure comprised of
15 regular components lends itself local interconnects, which minimizes parasitic capacitance and resistance, thus increasing circuit speed. A third objective of the present invention is, therefore, to provide a universal multiplier-accumulator having an array structure with a high degree of component regularity, in order to facilitate circuit
20 layout and minimize the length of interconnects.

Summary of the Invention

The invention is a circuit that can perform universal multiplication-accumulation. It is universal in the sense that it operates with equal facility on operands of
25 unsigned magnitude, operands in a two's complement mode, and operands of mixed modes.

By deriving special compensation expressions for terms having negative weight for each multiplication mode (i.e., two's complement, unsigned magnitude, and mixed modes), and
30 by grouping these compensations in a particular manner, it is possible to eliminate the need for the generation of

negative partial products. Further, by making these compensation expressions conformal across all multiplication modes, specialized circuitry is minimized and circuit regularity is maximized. Similar adder elements are arrayed, with all specialized circuitry being implemented as circuit modifications on some of these elements, thus improving circuit regularity and interconnectivity.

Fast adder elements are included to improve overall speed of the invention. Speed is further increased with the use of multiple-row addition techniques.

Brief Description of the Drawings

Figures 1a-1c depict multiplier input operands X and Y and accumulator input Z in binary form (e.g., in Figure 1a, $m=8$ for an 8-bit word, and each a is a bit in that word);

Figures 2a-2c depict expressions that yield magnitudes of two's complement terms X_{tc} , Y_{tc} , and Z_{tc} ;

Figures 2d-2f depict expressions that yield magnitudes of unsigned magnitude terms X_{uns} , Y_{uns} , and Z_{uns} ;

Figures 3a-3d depict expressions for the products $P_{tc} = X_{tc} Y_{tc}$, $P_{uns} = X_{uns} Y_{uns}$, $P_A = X_{tc} Y_{uns}$, and $P_B = X_{uns} Y_{tc}$, respectively;

Figures 4a-4e, 5a-5f, 6a-6f, and 7a-7e depict two-row compensation derivations for L , N , O , and Z , respectively;

Figures 8-11 depict multiplication-accumulation in unsigned mode, two's complement mode, and mixed modes A and B ("A" figures show a standard representation, "B" figures

4

show the representation condensed and slightly rearranged, and "C" figures show an array configuration that will perform the multiplication-accumulation);

5 Figure 12a is block diagram of the preferred multiplier-accumulator;

Figure 12b is a schematic of the array portion of the preferred multiplier-accumulator;

10 Figures 13-22 detail preferred adder elements and their operation ("A" figures are schematics of the elements, while "B" figures are logical symbols, which are used in Figure 12, for example);

Figure 23 depicts a logic diagram for a preferred Exclusive-OR (XOR) gate; and

15 Figure 24 depicts the logic circuitry which generates control signals TCA, TCB, TCC, and MXM.

Preferred Embodiment of the Invention

20 The preferred embodiment of the universal multiplier-accumulator makes use of certain algorithms, the understanding of which is essential to an understanding of the multiplier-accumulator. Therefore, a derivation of these algorithms follows.

1. Derivations

25 The universal multiplier-accumulator has two multiplier input operands X and Y (also called product terms) and one accumulator term Z, expressed in Figures 1a-1c as binary power expansions. The two possible accumulator input modes are two's complement and unsigned

magnitude. In this disclosure, subscript "tc" indicates two's complement mode, and subscript "uns" indicates unsigned magnitude mode. For example, X_{tc} indicates X interpreted as a two's complement number. Further, in this disclosure, a term may be represented as a vector. For example, X (shown as a binary power expansion in Figure 1) may also be represented as the vector $(a_{n-1}, a_{n-2}, \dots, a_0)$, Y as $(b_{n-1}, b_{n-2}, \dots, b_0)$, and Z as $(c_{m+n-1}, c_{m+n-2}, \dots, c_0)$. Multiplication is indicated by the symbol " \cdot " and addition is indicated by the symbol "+". Logic operations are indicated in capitals, such as in the expression " $a \text{ AND } b$ ", for example.

Figure 2a is a two's complement expression of X : when X is to be interpreted as X_{tc} , the expression in Figure 2a will yield the magnitude of X_{tc} . Figures 2b-2c show corresponding expressions for Y_{tc} and Z_{tc} . Figures 2d-2f show similar expressions for X_{uns} , Y_{uns} , and Z_{uns} .

Since each of the input operands X and Y can be in two's complement mode or unsigned magnitude mode, there are four possible multiplication modes for the operation $P=X \cdot Y$. These are two's complement mode ($P_{tc}=X_{tc} \cdot Y_{tc}$), unsigned magnitude mode ($P_{uns}=X_{uns} \cdot Y_{uns}$), mixed mode A ($P_A=X_{tc} \cdot Y_{uns}$), and mixed mode B ($P_B=X_{uns} \cdot Y_{tc}$). Figures 3a-3d, respectively, show expressions for each of these multiplication modes, based on Figures 2a-2f.

Note in Figures 3a-3d that each product includes different sign combinations of expressions L , M , N , and O . That is, $P_{tc}=+L+M-N-O$, $P_{uns}=+L+M+N+O$, $P_A=-L+M+N-O$, and $P_B=-L+M-N+O$. This is a significant observation and has bearing on the design of the preferred embodiment.

It is well known that binary addition of a two's complement form of a word is equivalent to its unsigned binary subtraction. Addition is more straightforward and

readily implemented than subtraction. To obtain a two's complement form, a word is inverted and a 1 is added to it. In the above-listed multiplication modes, terms L , N , and O are subtracted. In the preferred embodiment, "two-row compensations" are derived to aid in these subtractions. The two-row compensation L_{2rc} , for example, is a pair of rows of bits that when summed with an operand, gives a final accumulation that is as if L was subtracted. The two-row compensation contains two's complement arithmetic in a form that allows efficient design of the subject multiplier-accumulator.

Figure 4a shows the expression for term L , written the same as in Figure 3a, for example. L may also be expressed as a summation of the two rows in Figure 4b. Figure 4c shows L inverted and a 1 added to obtain a two's complement inversion, rewritten in Figure 4d. Figure 4e shows L in a two's complement binary matrix form. Figure 4e is the two-row compensation L_{2rc} .

Two-row compensation N_{2rc} is generated as shown in Figures 5a-5f. Figure 5a expresses term N , the same as in Figure 3b, for example. N may also be expressed as shown in Figure 5b. Note that b_{n-1} is a binary constant, a 1 or 0. If $b_{n-1}=1$, then N reduces to Figure 5c. If $b_{n-1}=0$, then N and its two's complement are both zero. For $b_{n-1}=1$, the two's complement of N is shown in Figure 5d. Using this information, a generalized expression for the two's complement of N can be created, shown in Figure 5e. The expression of Figure 5e is then converted to N_{2rc} , shown in Figure 5f.

Term O (Figure 6a) is similar to N . The derivation of O_{2rc} , shown in Figure 6f, is therefore derived using a similar technique.

Since the invention accumulates as well as multiplies, two-row compensation Z_{2rc} for accumulator term Z must also be derived, since Z may be positive or negative in any multiplication-accumulation mode involving two's complement. This is shown in Figures 7a-7e. Note that when $c_{m-1}=1$, $Z<0$, and when $c_{m-1}=0$, $Z>0$.

With reference to Figures 8-11, the four multiply-accumulate modes are now detailed for 8-bit X and Y terms ($m=8$, $n=8$). In each figure, a somewhat standard representation of the operation is shown, and then an equivalent, preferred arrangement. The preferred arrangements are readily adaptable to embodiment in a single circuit. M remains the same across all cases.

Figure 8a illustrates an unsigned mode multiplication-accumulation, that is, $X_{uns} Y_{uns} + Z_{uns}$. Elements of L , M , N and O are shown in their proper columns for addition. Figure 8b shows the same algorithm, but with O moved over the upper left edge of M . Figure 8c shows this operation in an array configuration. The correct result is obtained if addition is diagonally performed.

Figure 9a illustrates a two's complement mode multiplication-accumulation, that is, $X_{tc} Y_{tc} + Z_{tc}$. Recall from Figure 3a that N and O are subtracted in this mode. Instead of subtracting N and O in the preferred embodiment, two-row compensations N_{2rc} and O_{2rc} are summed. Because Z may be positive or negative in this mode, Z_{2rc} is also summed. Figure 9b shows the same derivation, but with all ones in the two-row compensations pre-summed and all but two elements of O_{2rc} moved over the upper left edge of M . For the specific case of a multiply-only algorithm (no accumulation), the inventive method reduces to the familiar Baugh-Wooley two's complement multiplier method. Figure 9c shows the operation in an array configuration. The correct result is obtained if addition is performed diagonally.

Mixed mode A ($X_{tc} Y_{uns} + Z_{tc}$) is illustrated in Figure 10a. As shown in Figure 3c, L and O are subtracted in this mode. Therefore L_{2rc} and O_{2rc} are added. Z_{2rc} is also added because Z may be positive or negative in this mode. Figure 10b shows the corresponding preferred arrangement. As in two's complement mode in Figure 9b, all ones in the two-row compensations are pre-summed. As above, Figure 10c shows an array configuration.

Finally, Figure 11a illustrates mixed mode B ($X_{uns} Y_{tc} + Z_{tc}$). As shown in Figure 3d, L and N are subtracted in this mode. Therefore L_{2rc} and N_{2rc} are added. Z_{2rc} is also added because Z may be positive or negative in this mode. Figure 11c shows a corresponding array configuration.

2. Embodiment

The preferred embodiment of the invention includes an n by m multiplier array having multiple-function adder elements to perform the above-derived algorithms as represented in the array configurations of Figures 8c, 9c, 10c, and 11c. For the sake of illustration, an 8 by 8 multiplier-accumulator according to the invention is shown in Figures 12a and 12b. The preferred embodiment merges the four multiplication-accumulation modes into a single arrayed set 10 of adder elements 100-107, 110-117, 120-127, 130-137, 140-147, 150-157, 160-169, 170-179, and 180-188, detailed in Figures 13 through 23, of which the "A" figures show a schematic, and the "B" figures show a corresponding logical symbol used in Figure 12b. It is noted that some of the signals shown in the logical symbols are to be interpreted differentially, and some nondifferentially. For example, SIN in Figure 14b is differential (representing true SIN and complement SIN*, shown in Figure 14a), while input a_i is not.

The elements will now be described. The following chart is provided for cross-reference:

Adder			
	Type	Figure	Elements
5	-----		
	FA18	13	110, 120, 130, 140, 150, 160, 170, 172-177, 180-187
	FA1A	14	111-116, 121-126, 131-136, 141-146, 151-156, 178
10	FA2A	15	100-106, 171
	FAC	16	161-166
	FACA	17	107, 117, 127, 137, 147, 157
	FACC	18	167
	FAAC	19	168
15	FAAC3	20	169
	HAC	21	179
	HAC2	22	188

FA18 (Figure 13) is a 3-bit fully differential full-adder cell, with propagation times of 2 gate delays from sum-in to sum-out and 1 gate delay from carry-in to carry-out, when the preferred EXOR circuit of Figure 23 is used.

FA1A (Figure 14) performs:
 $(a_i, b_j) + \text{SIN} + \text{CIN}.$

FA2A (Figure 15) performs:
 $(a_{j+1}, b_0) + (a_j, b_1) + \text{CIN}.$

FAC (Figure 16) performs:
 $((a_{n-j}, b_{n-1} \text{ AND } \text{TCB}) \text{ OR } (a_{n-j}, b_{n-1} \text{ AND } \text{TCB}^*)) + \text{SIN} + \text{CIN},$ where TCB^* is the inverse of $\text{TCB}.$

FACA (Figure 17) performs:
 $((a_{n-1}, b_{n-j} \text{ AND } \text{TCA}) \text{ OR } (a_{n-1}, b_{n-j} \text{ AND } \text{TCA}^*)) + a_{n-2}, b_{n-j+1} + \text{CIN}.$

FACC (Figure 18) performs:

$$((a_{n-1} b_{n-2} \text{ AND } TCA) \text{ OR } (a_{n-1} b_{n-2} \text{ AND } TCA*)) + \\ ((a_{n-2} b_{n-1} \text{ AND } TCB) \text{ OR } (a_{n-2} b_{n-1} \text{ AND } TCB*)) + CIN.$$

FAAC (Figure 19) performs:

$$((a_{n-1} b_{n-1} \text{ AND } MXM) \text{ OR } (\text{NOT}(a_{n-1} b_{n-1}) \text{ AND } MXM*)) + \\ a_{n-1} TCA + b_{n-1} TCB. \text{ Note here that } A B+1 \text{ is logically} \\ \text{equivalent to } \text{NOT}(A B).$$

FAAC3 (Figure 20) performs:

$$((c_{n+n-1} \text{ AND } TCC) \text{ OR } (c_{n+n-1} \text{ AND } TCC*)) + A_{n-1} TCA + B_{n-1} TCB.$$

HAC (Figure 21) performs: $TCC + SIN.$

HAC2 (Figure 22) performs: $A + B.$

FAC, FACA, FACC, FAAC, FAAC3 are all adder elements that perform more than one type of addition, and are controlled by signals TCA, TCB, TCC, and MXM. The generation of TCA, TCB, TCC, and MXM is detailed in Figure 24 and in the chart below.

Mode	TCA	TCB	TCC	MXM
-----	-----	-----	-----	-----
Unsigned	0	0	0	0
Mixed Mode B	0	1	0	1
Mixed Mode A	1	0	0	1
Two's Complement	1	1	1	0

The above elements when connected as shown in Figure 12 perform together as a universal multiplier-accumulator, capable of multiplying in unsigned mode, two's complement mode, and mixed modes A and B.

Figure 12b further shows the preferred embodiment having multiple data input bits V_{ss} (logical 0), V_{DD} (logical 1), a_0 - a_7 , b_0 - b_7 , and carry in bits c_0 - c_{15} . Output bits

include carry-out bits c_0-c_{15} and sum bits s_0-s_{16} . MXM , TCA , TCB , and TCC are control inputs, and NC denotes a no-connect. Sum-out and carry-out bits s_0-s_{16} and co_0-co_{15} are summed in final adder 11 of Figure 12a.

5 Simplicity and interconnectivity are improved by designing similarity across all elements. For example, each full-adder shares the same basic primitive cell (FA18) with compensation logic included as required. Some adder elements must be able to perform more than one function.
10 It is primarily for this reason that several adder types are used in the preferred embodiment.

For example, element 168 of Figure 12b multiply-accumulates $a_7 b_7$ in unsigned mode, $a_7 b_7 + a_7 + b_7$ in two's complement mode, $A_7 B_7 + a_7$ in mixed mode A, and $A_7 B_7 + b_7$ in
15 mixed mode B (compare Figures 8c, 9c, 10c, and 11c, respectively).

The partial products within term M for all multiplication modes are performed within elements 100-106, 200-206, 300-306, 400-406, 500-506, 600-606, and 700-
20 706. For example, the partial product $a_0 b_0$ is implemented by FA2A element 100 when connected as shown in Figure 12b.

Term O of Figure 8a is shown in Figure 8b at the upper left side of M. These terms are implemented by elements 107, 117, 127, 137, 147, 157, and 167. For example, term
25 $a_7 b_0$ in Figure 8b is implemented by element 107 when connected as shown.

Term N of Figures 8a and 8b is implemented by elements 160-166. For example, term $a_0 b_7$ is implemented by element 160 when connected as shown.

30 These terms in other modes are similarly accomplished by the preferred embodiment.

Elements 108 and 171 are now further elucidated. Term a_{m-1} is added in the column of c_{m-1} in two's complement mode and mixed mode A. Term b_{n-1} is added in the column of c_{n-1} in two's complement mode and mixed mode B. In the preferred embodiment, $m=n=8$, so a_7 and/or b_7 are added in the column of c_7 in all but unsigned mode. The addition of term a_7 is accomplished by element 108. Note that 108 outputs a differential signal. The addition of term b_7 is accomplished by element 171. This arrangement allows proper addition of a_{m-1} and b_{n-1} even if $m \neq n$.

Final addition of the sumout and carryout terms of the preferred array 10 is performed by final adder 11, shown in Figure 12a, which comprises any of several possible adder configurations, including, for example, full carry look ahead, carry select, and conditional-sum type adders.

The preferred embodiment is fast when fast adders (such as the preferred) are used. Speed is further improved in the preferred embodiment in Figure 12 by summing all even rows together, summing all odd rows together, and adding the even sum with the odd sum in final adder 11 (a Wallace tree technique). It is noted that three or more groups of rows can similarly be summed, but the apparent speed improvement thus gained is lost due to increased parasitic capacitances in the longer interconnections. A path limit of 22 mils or less is deemed desirable to limit parasitics and optimize speed. Short interconnections therefore makes Wallace tree summation of two groups of rows preferable.

Many variations may be made to the embodiment without making it a different invention. Different adder element designs may be used, as well as alternate EXOR designs. Although the preferred embodiment is integrated into a semiconductor substrate using CMOS techniques, other fabrication technologies might be used. The circuit may be constructed discretely and still embody the same invention.

Claims:

1. Apparatus to obtain a product P from terms X and Y and accumulate it with a term Z , the result for use in a digital processing system, wherein P is one of the group
 5 $P_{tc} = X_{tc} Y_{tc}$, $P_{uns} = X_{uns} Y_{uns}$, $P_A = X_{tc} Y_{uns}$, and $P_B = X_{uns} Y_{tc}$, wherein X is representable as vector $(a_{m-1}, a_{m-2}, \dots, a_0)$ and Y as vector $(b_{n-1}, b_{n-2}, \dots, b_0)$, comprising:

L means, to form $L = a_{m-1} b_{n-1} 2^{m+n-2}$;

10 M means, to form $M = \sum \sum a_i b_j 2^{i+j}$ for i from 0 to $m-2$ and j from 0 to $n-2$;

N means, to form $N = \sum a_i b_{n-1} 2^{n-1+i}$ for i from 0 to $m-2$;

O means, to form $O = \sum a_{m-1} b_j 2^{m-1+j}$ for j from 0 to $n-2$;

15 compensation means, to form one or more compensations L_{2rc} , N_{2rc} , and O_{2rc} , in the event that one or more of L , N , and O , respectively, are negative;

20 P means, to form P of one of the group P_{uns} , P_{tc} , P_A , and P_B , wherein P_{uns} is formed by summing L , M , N , and O , P_{tc} is formed by summing L , M , N_{2rc} , and O_{2rc} , P_A is formed by summing L_{2rc} , M , N , and O_{2rc} , and P_B is formed by summing L_{2rc} , M , N_{2rc} , and O ; and

accumulation means, to accumulate P with Z ;

wherein all summations are done in an interconnected array of substantially identical 3-bit adders arranged in a Wallace tree.

5 2. Apparatus to form a product P of $X=(a_{m-1}, a_{m-2}, \dots, a_0)$ and $Y=(b_{n-1}, b_{n-2}, \dots, b_0)$, and accumulate P with a term Z , wherein the result is for use in a digital processing system, and wherein each of X and Y may be in two's complement mode or in unsigned mode, comprising:

10 first means, to form terms $a_i b_j$ for all i and j yielding positive values;

second means, to form a combination of positive terms equivalent to any combination of terms $a_i b_j$ which is negative;

15 third means, to selectively combine said terms formed by said first means and said combination of positive terms formed by said second means, to form P ; and

accumulation means, to accumulate P with Z ;

wherein all summations are done in an interconnected array of substantially identical 3-bit adders arranged in a Wallace tree.

3. Apparatus to form a product $P=(p_{m+n-1}, p_{m+n-2}, \dots, p_0)$ of
 5 $X=(a_{m-1}, a_{m-2}, \dots, a_0)$ and $Y=(b_{n-1}, b_{n-2}, \dots, b_0)$, and accumulate P with a term Z , wherein the result is for use in a digital processing system, and wherein each of X and Y may be in two's complement mode or in unsigned mode, comprising:

means to complement terms a_i and b_j to obtain terms A_i and
 10 B_j , respectively, for $0 \leq i \leq m-1$ and $0 \leq j \leq n-1$;

means to form terms $a_{m-1} b_{n-1}$, $a_i b_j$, $a_i b_{n-1}$, $A_i b_{n-1}$, $a_{m-1} b_j$, and $a_{m-1} B_j$, for $0 \leq i \leq m-2$ and $0 \leq j \leq n-2$;

means to form a term which is the complement of said term $a_{m-1} b_{n-1}$;

15 means to form at least one term having the constant value 1; and

means to selectively combine said terms so formed, including:

17

means to form p_k for $0 \leq k \leq m+n-2$, to form P for X and Y both in unsigned mode, by summing said terms $a_i b_j$ where $i+j=k$, and carries from the sum for $k-1$;

5

means to form p_k for $0 \leq k \leq m+n-1$, to form P for X and Y both in two's complement mode,

by adding to the sum for each k for $0 \leq i \leq m-2$ and $0 \leq j \leq n-2$ said terms $a_i b_j$, $a_{m-1} b_{n-1}$, $A_i b_{n-1}$, $a_{m-1} B_j$, and $A_{m-1} B_{n-1}$, wherein the sum of the subscripts within each term is equal to k ,

10

by adding said terms a_{m-1} and b_{n-1} to the sum for $k=m+n-2$,

by adding said terms A_{m-1} , B_{n-1} , and 1 to the sum for $k=m+n-1$, and

15

by adding carries from the sum for each $k-1$ to the sum for each k ;

means to form p_k for $0 \leq k \leq m+n-1$, to form P for X in two's complement mode and Y in unsigned mode,

20

by adding to the sum for each k for $0 \leq i \leq m-2$ and $0 \leq j \leq n-2$ said terms $a_i b_j$, $a_{m-1} b_{n-1}$, $A_i b_{n-1}$, $a_{m-1} B_j$, and $A_{m-1} B_{n-1}$, wherein the sum of the subscripts within each term is equal to k ,

18

by adding said terms a_{m-1} and 1 to the sum for $k=m+n-2$,

by adding said terms A_{m-1} and 1 to the sum for $k=m+n-1$, and

5 by adding carries from the sum for each $k-1$ to the sum for each k ;

means to form p_k for $0 \leq k \leq m+n-1$, to form P for X in unsigned mode and Y in two's complement mode,

10 by adding to the sum for each k for $0 \leq i \leq m-2$ and $0 \leq j \leq n-2$ said terms $a_i, b_j, b_{n-1}, A_i, b_{n-1}, a_{m-1}, b_j$, and A_{m-1}, B_{n-1} , wherein the sum of the subscripts within each term is equal to k ,

by adding said terms b_{n-1} and 1 to the sum for $k=m+n-2$,

15 by adding said terms B_{n-1} and 1 to the sum for $k=m+n-1$, and

by adding carries from the sum for each $k-1$ to the sum for each k ; and

accumulation means, to accumulate P with Z ;

wherein all summations are done in an interconnected array of substantially identical 3-bit adders arranged in a Wallace tree.

4. Apparatus to obtain a product P and accumulate it with a term Z, the result for use in a digital processing system, wherein P is one of the group $P_{tc}=X_{tc} Y_{tc}$, $P_{uns}=X_{uns} Y_{uns}$, $P_A=X_{tc} Y_{uns}$, and $P_B=X_{uns} Y_{tc}$, wherein X is representable as vector $(a_{m-1}, a_{m-2}, \dots, a_0)$ and Y as vector $(b_{n-1}, b_{n-2}, \dots, b_0)$, comprising:

10 first means, to form

$$\begin{aligned} L &= a_{m-1} b_{n-1} 2^{m+n-2}, \\ M &= \sum \sum a_i b_j 2^{i+j} \text{ for } i \text{ from } 0 \text{ to } m-2 \text{ and } j \text{ from } 0 \text{ to } n-2, \\ N &= \sum a_i b_{n-1} 2^{n-1+i} \text{ for } i \text{ from } 0 \text{ to } m-2, \text{ and} \\ 15 \quad O &= \sum a_{m-1} b_j 2^{m-1+j} \text{ for } j \text{ from } 0 \text{ to } n-2; \end{aligned}$$

compensation means, to form one or more compensations L_{2rc} , N_{2rc} , and O_{2rc} , in the event that one or more of L, N, and O, respectively, are negative; and

20 second means, to form P of one of the group P_{uns} , P_{tc} , P_A , and P_B , wherein P_{uns} is formed by summing L, M, N, and O, P_{tc} is formed by summing L, M, N_{2rc} , and O_{2rc} , P_A is formed by

20

summing L_{2rc} , M , N , and O_{2rc} , and P_B is formed by summing L_{2rc} , M , N_{2rc} , and ;

third means to select one of the group P_{uns} , P_{tc} , P_A , and P_B , to provide as P ; and

5 accumulation means, to accumulate P with Z ;

wherein all summations are done in an interconnected array of substantially identical 3-bit adders arranged in a Wallace tree.

10 5. Apparatus to obtain a product P for use in a digital processing system, wherein P is one of the group $P_{tc} = X_{tc} Y_{tc}$, $P_{uns} = X_{uns} Y_{uns}$, $P_A = X_{tc} Y_{uns}$, and $P_B = X_{uns} Y_{tc}$, wherein X is representable as vector $(a_{m-1}, a_{m-2}, \dots, a_0)$ and Y as vector $(b_{n-1}, b_{n-2}, \dots, b_0)$, comprising:

L means, to form $L = a_{m-1} b_{n-1} 2^{m+n-2}$;

15 M means, to form $M = \sum a_i b_j 2^{i+j}$ for i from 0 to $m-2$ and j from 0 to $n-2$;

N means, to form $N = \sum a_i b_{n-1} 2^{n-1+i}$ for i from 0 to $m-2$;

O means, to form $O = \sum a_{m-1} b_j 2^{m-1+j}$ for j from 0 to $n-2$; and

P means, to form P of one of the group P_{uns} , P_{tc} , P_A , and P_B , wherein P_{uns} is formed by summing L, M, N, and O, P_{tc} is formed by summing L and M and subtracting N and O, P_A is formed by summing M and N and subtracting L and O, and P_B is formed by summing M and O and subtracting L and N.

6. The apparatus of Claim 5, further comprising means within each of said L, N, and O means, to form compensations L_{2rc} , N_{2rc} , and O_{2rc} , respectively, so that any subtraction of L, N, or O within said P means is obtained by a summation of L_{2rc} , N_{2rc} , or O_{2rc} , respectively.

7. Apparatus to obtain a product P for use in a digital processing system, wherein P is one of the group $P_{tc}=X_{tc} Y_{tc}$, $P_{uns}=X_{uns} Y_{uns}$, $P_A=X_{tc} Y_{uns}$, and $P_B=X_{uns} Y_{tc}$, wherein X is representable as vector $(a_{m-1}, a_{m-2}, \dots, a_0)$ and Y as vector $(b_{n-1}, b_{n-2}, \dots, b_0)$, comprising:

L means, to form $L = a_{m-1} b_{n-1} 2^{m+n-2}$;

M means, to form $M = \sum_i a_i b_j 2^{i+j}$ for i from 0 to m-2 and j from 0 to n-2;

N means, to form $N = \sum_i a_i b_{n-1} 2^{n-1+i}$ for i from 0 to m-2;

O means, to form $O = \sum_j a_{m-1} b_j 2^{m-1+j}$ for j from 0 to n-2;

22

compensation means, to form one or more compensations L_{2RC} , N_{2RC} , and O_{2RC} , in the event that one or more of L , N , and O , respectively, are negative; and

5 P means, to form P of one of the group P_{uns} , P_{tc} , P_A , and P_B , wherein P_{uns} is formed by summing L , M , N , and O , P_{tc} is formed by summing L , M , N_{2RC} , and O_{2RC} , P_A is formed by summing L_{2RC} , M , N , and O_{2RC} , and P_B is formed by summing L_{2RC} , M , N_{2RC} , and O .

8. Apparatus to form a product P of $X=(a_{n-1}, a_{n-2}, \dots, a_0)$ and
10 $Y=(b_{n-1}, b_{n-2}, \dots, b_0)$, wherein P is for use in a digital processing system, and wherein each of X and Y may be in two's complement mode or in unsigned mode, comprising:

first means, to form terms $a_i b_j$ for all i and j yielding positive values;

15 second means, to form a combination of positive terms equivalent to any combination of terms $a_i b_j$ which is negative; and

third means, to selectively combine said terms formed by said first means and said combination of positive terms
20 formed by said second means, to form P .

9. Apparatus to form a product $P = (P_{m+n-1}, P_{m+n-2}, \dots, P_0)$ of $X = (a_{m-1}, a_{m-2}, \dots, a_0)$ and $Y = (b_{n-1}, b_{n-2}, \dots, b_0)$, wherein P is for use in a digital processing system, and wherein each of X and Y may be in two's complement mode or in unsigned mode, comprising:

means to complement terms a_i and b_j to obtain terms A_i and B_j , respectively, for $0 \leq i \leq m-1$ and $0 \leq j \leq n-1$;

means to form terms $a_{m-1} b_{n-1}$, $a_i b_j$, $a_i b_{n-1}$, $A_i b_{n-1}$, $a_{m-1} b_j$, and $a_{m-1} B_j$, for $0 \leq i \leq m-2$ and $0 \leq j \leq n-2$;

10 means to form a term which is the complement of said term $a_{m-1} b_{n-1}$;

means to form at least one term having the constant value 1; and

means to selectively combine said terms so formed.

15 10. The apparatus of Claim 9, wherein, for X and Y both in unsigned mode, said means for selectively combining includes:

means to form p_k for $0 \leq k \leq m+n-2$, by summing said terms $a_i b_j$ where $i+j=k$, and carries from the sum for $k-1$.

11. The apparatus of Claim 9, wherein, for X and Y both in two's complement mode, said means for selectively combining includes:

means to form p_k for $0 \leq k \leq m+n-1$,

5 by adding to the sum for each k for $0 \leq i \leq m-2$ and $0 \leq j \leq n-2$ said terms $a_i b_j$, $a_{m-1} b_{n-1}$, $A_i b_{n-1}$, $a_{m-1} B_j$, and $a_{m-1} b_{n-1}$, wherein the sum of the subscripts within each term is equal to k,

10 by adding said terms a_{m-1} and b_{n-1} to the sum for $k=m+n-2$,

by adding said terms A_{m-1} , B_{n-1} , and 1 to the sum for $k=m+n-1$, and

by adding carries from the sum for each k-1 to the sum for each k.

15 12. The apparatus of Claim 9, wherein, for X in two's complement mode and Y in unsigned mode, said means for selectively combining includes:

means to form p_k for $0 \leq k \leq m+n-1$,

25

by adding to the sum for each k for $0 \leq i \leq m-2$ and $0 \leq j \leq n-2$ said terms $a_i b_j$, $a_{m-1} b_{n-1}$, $a_{m-1} b_j$, and $a_i b_{n-1}$, wherein the sum of the subscripts within each term is equal to k ,

5 by adding said terms a_{m-1} and 1 to the sum for $k=m+n-2$,

by adding said terms A_{m-1} and 1 to the sum for $k=m+n-1$; and

by adding carries from the sum for each $k-1$ to the sum for each k .

10 13. The apparatus of Claim 9, wherein, for X in unsigned mode and Y in two's complement mode, said means for selectively combining includes:

means to form p_k for $0 \leq k \leq m+n-1$,

15 by adding to the sum for each k for $0 \leq i \leq m-2$ and $0 \leq j \leq n-2$ said terms $a_i b_j$, b_{n-1} , $A_i b_{n-1}$, $a_{m-1} b_j$, and $A_{m-1} b_{n-1}$, wherein the sum of the subscripts within each term is equal to k ,

by adding said terms b_{n-1} and 1 to the sum for $k=m+n-2$,

by adding said terms B_{n-1} ²⁶ and 1 to the sum for $k=m+n-1$; and

by adding carries from the sum for each $k-1$ to the sum for each k .

5 14. The apparatus of Claim 9, wherein said means for selectively combining includes:

means to form p_k for $0 \leq k \leq m+n-2$, to form P for X and Y both in unsigned mode, by summing said terms $a_i b_j$ where $i+j=k$, and carries from the sum for $k-1$;

10 means to form p_k for $0 \leq k \leq m+n-1$, to form P for X and Y both in two's complement mode,

by adding to the sum for each k for $0 \leq i \leq m-2$ and $0 \leq j \leq n-2$ said terms $a_i b_j$, $a_{m-1} b_{n-1}$, $A_i b_{n-1}$, $a_{m-1} B_j$, and $a_{m-1} b_{n-1}$, wherein the sum of the subscripts within each term is equal to k ,

15 by adding said terms a_{m-1} and b_{n-1} to the sum for $k=m+n-2$,

by adding said terms A_{m-1} , B_{n-1} , and 1 to the sum for $k=m+n-1$, and

27

by adding carries from the sum for each $k-1$ to the sum for each k ;

means to form p_k for $0 \leq k \leq m+n-1$, to form P for X in two's complement mode and Y in unsigned mode,

5 by adding to the sum for each k for $0 \leq i \leq m-2$ and $0 \leq j \leq n-2$ said terms $a_i b_j$, $a_{m-1} b_{n-1}$, $a_{m-1} b_j$, and $a_i b_{n-1}$, wherein the sum of the subscripts within each term is equal to k ,

by adding said terms a_{m-1} and 1 to the sum for $k=m+n-2$,

10 by adding said terms a_{m-1} and 1 to the sum for $k=m+n-1$, and

by adding carries from the sum for each $k-1$ to the sum for each k ;

15 means to form p_k for $0 \leq k \leq m+n-1$, to form P for X in unsigned mode and Y in two's complement mode,

by adding to the sum for each k for $0 \leq i \leq m-2$ and $0 \leq j \leq n-2$ said terms $a_i b_j$, b_{n-1} , $a_i b_{n-1}$, $a_{m-1} b_j$, and $a_{m-1} b_{n-1}$, wherein the sum of the subscripts within each term is equal to k ,

20 by adding said terms b_{n-1} and 1 to the sum for $k=m+n-2$,

28

by adding said terms B_{n-1} and 1 to the sum for $k=m+n-1$; and

by adding carries from the sum for each $k-1$ to the sum for each k .

5 15. Apparatus to obtain a product P for use in a digital processing system, wherein P is one of the group $P_{tc}=X_{tc} Y_{tc}$, $P_{uns}=X_{uns} Y_{uns}$, $P_A=X_{tc} Y_{uns}$, and $P_B=X_{uns} Y_{tc}$, wherein X is representable as vector $(a_{m-1}, a_{m-2}, \dots, a_0)$ and Y as vector $(b_{n-1}, b_{n-2}, \dots, b_0)$, comprising:

10 first means, to form

$$L = a_{m-1} b_{n-1} 2^{m+n-2},$$

$$M = \sum \sum a_i b_j 2^{i+j} \text{ for } i \text{ from } 0 \text{ to } m-2 \text{ and } j \text{ from } 0 \text{ to } n-2,$$

$$N = \sum a_i b_{n-1} 2^{n-1+i} \text{ for } i \text{ from } 0 \text{ to } m-2, \text{ and}$$

15 $O = \sum a_{m-1} b_j 2^{m-1+j} \text{ for } j \text{ from } 0 \text{ to } n-2;$

second means, to form P_{uns} , P_{tc} , P_A , and P_B , wherein P_{uns} is formed by summing L , M , N , and O , P_{tc} is formed by summing L and M and subtracting N and O , P_A is formed by summing M and N and subtracting L and O , and P_B is formed by summing M and O and subtracting L and N ; and

20

third means, to select one of the group P_{uns} , P_{tc} , P_A , and P_B ,
to provide as P .

16. The apparatus of Claim 15, which further comprises
means within each of said L , N , and O means, to form
5 compensations L_{2RC} , N_{2RC} , and O_{2RC} , respectively, so that any
subtraction of L , N , or O within said second means is
obtained by a summation of L_{2RC} , N_{2RC} , or O_{2RC} , respectively.

17. Apparatus to obtain a product P for use in a digital
processing system, wherein P is one of the group $P_{tc} = X_{tc} Y_{tc}$,
10 $P_{uns} = X_{uns} Y_{uns}$, $P_A = X_{tc} Y_{uns}$, and $P_B = X_{uns} Y_{tc}$, wherein X is
representable as vector $(a_{m-1}, a_{m-2}, \dots, a_0)$ and Y as vector
 $(b_{n-1}, b_{n-2}, \dots, b_0)$, comprising:

first means, to form

$$\begin{aligned} L &= a_{m-1} b_{n-1} 2^{m+n-2}, \\ 15 \quad M &= \sum \sum a_i b_j 2^{i+j} \text{ for } i \text{ from } 0 \text{ to } m-2 \text{ and } j \text{ from } 0 \text{ to } \\ &\quad n-2, \\ N &= \sum a_i b_{n-1} 2^{n-1+i} \text{ for } i \text{ from } 0 \text{ to } m-2, \text{ and} \\ O &= \sum a_{m-1} b_j 2^{m-1+j} \text{ for } j \text{ from } 0 \text{ to } n-2; \end{aligned}$$

compensation means, to form one or more compensations L_{2RC} ,
20 N_{2RC} , and O_{2RC} , in the event that one or more of L , N , and O ,
respectively, are negative; and

second means, to form P of one of the group P_{uns} , P_{tc} , P_A , and P_B , wherein P_{uns} is formed by summing L , M , N , and O , P_{tc} is formed by summing L , M , N_{2rc} , and O_{2rc} , P_A is formed by summing L_{2rc} , M , N , and O_{2rc} , and P_B is formed by summing L_{2rc} , M , N_{2rc} , and O ; and

third means to select one of the group P_{uns} , P_{tc} , P_A , and P_B , to provide as P .

18. The apparatus of Claim 17, which further comprises accumulation means, to accumulate P with an accumulation term Z .

19. The apparatus of Claims 1, 2, 3, 4, 5, 7, 8, 14, 15 or 17, wherein at least one of said substantially identical 3-bit adders includes:

a sum generator, comprising:

15 first and second XOR means;

first and second inputs of said first XOR means being differentially responsive to first and second bits, respectively, and first and second inputs of said second XOR means being differentially responsive to an

output of said first XOR means³¹, and a third bit,
respectively;

5 wherein an output of said second XOR means is able to
provide a differential output for the sum generator,
having a first state when an odd subset of said first,
second, and third bits are true, and having a second
state when an even subset of said first, second, and
third bits are true;

10 wherein at least one of said first and second XOR
means comprises:

first, second, third, and fourth tristate means;

an input of each of said tristate means being
inputs W, X, Y, and Z, respectively, of the XOR
means;

15 an output of each of said first and second
tristate means being connected together to form
a first XOR means output;

20 an output of each of said third and fourth
tristate means being connected together to form
a second XOR means output; and

32

said first and third tristate means being
tristated when an enable input is in a first
state, and said second and fourth tristate means
being tristated when said enable input is in a
5 second state;

wherein said inputs W and Z are responsive
together and said inputs X and Y are responsive
together, said inputs W and X are differential-
ly responsive to a bit A, said enable input is
10 differentially responsive to a bit B, and said
first and second logic circuit outputs are able
to differentially provide an XOR function of said
bits A and B;

a carry-out generator, comprising:

15 a first input, responsive to said output of said first
XOR means;

a second input, differentially responsive to one of
said first and second bits;

a third input, differentially responsive to said third
20 bit; and

logic means, able to provide a first differential
state when at least two of said first, second, and

33

third bits are true, and otherwise able to provide a second state;

wherein said sum generator and said carry-out generator are able to provide a sum bit and a carry-out bit, respectively, on said first, second, and third bits;

wherein said logic means comprises:

first, second, third, and fourth tristate means;

an input of each of said tristate means being inputs W, X, Y, and Z, respectively, of the logic means;

an output of each of said first and second tristate means being connected together to form a first logic means output;

an output of each of said third and fourth tristate means being connected together to form a second logic means output; and

said first and third tristate means being tristated when an enable input is in a first state, and said second and fourth tristate means

34

being tristated when said enable input is in a second state;

wherein said enable input is said first input, said inputs X and Z form said second input, and said inputs W and Y form said third input;

wherein said sum generator and said carry-out generator are able to provide a sum bit and a carry-out bit, respectively, on said first, second, and third bits.

20. The apparatus of Claims 5, 7, 8, 9, or 15, which further comprises accumulation means, to accumulate P with an accumulation term Z.

21. The apparatus of Claims 5, 7, 8, 9, 15, or 17, wherein all summations are done in an interconnected array of substantially identical adders.

22. The apparatus of Claims 5, 7, 8, 9, 15, or 17, wherein said substantially identical adders comprise 3-bit adders.

23. The apparatus of Claims 5, 7, 8, 9, 15 or 17, wherein all summations are done using a Wallace tree technique.

24. A multiplier-accumulator array, having adders comprising:

a sum generator, comprising:

first and second XOR means;

5 first and second inputs of said first XOR means being differentially responsive to first and second bits, respectively, and first and second inputs of said second XOR means being differentially responsive to an output of said first XOR means, and a third bit,
10 respectively;

wherein an output of said second XOR means is able to provide a differential output for the sum generator, having a first state when an odd subset of said first, second, and third bits are true, and having a second
15 state when an even subset of said first, second, and third bits are true;

wherein at least one of said first and second XOR means comprises:

first, second, third, and fourth tristate means;

36

an input of each of said tristate means being inputs W, X, Y, and Z, respectively, of the XOR means;

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an output of each of said first and second tristate means being connected together to form a first XOR means output;

an output of each of said third and fourth tristate means being connected together to form a second XOR means output; and

10

said first and third tristate means being tristated when an enable input is in a first state, and said second and fourth tristate means being tristated when said enable input is in a second state;

15

wherein said inputs W and Z are responsive together and said inputs X and Y are responsive together, said inputs W and X are differentially responsive to a bit A, said enable input is differentially responsive to a bit B, and said first and second logic circuit outputs are able to differentially provide an XOR function of said bits A and B;

20

a carry-out generator, comprising:

37

a first input, responsive to said output of said first XOR means;

a second input, differentially responsive to one of said first and second bits;

5 a third input, differentially responsive to said third bit; and

logic means, able to provide a first differential state when at least two of said first, second, and third bits are true, and otherwise able to provide a second state;

10

wherein said sum generator and said carry-out generator are able to provide a sum bit and a carry-out bit, respectively, on said first, second, and third bits;

15 wherein said logic means comprises:

first, second, third, and fourth tristate means;

an input of each of said tristate means being inputs W, X, Y, and Z, respectively, of the logic means;

38

an output of each of said first and second tristate means being connected together to form a first logic means output;

5

an output of each of said third and fourth tristate means being connected together to form a second logic means output; and

10

said first and third tristate means being tristated when an enable input is in a first state, and said second and fourth tristate means being tristated when said enable input is in a second state;

wherein said enable input is said first input, said inputs X and Z form said second input, and said inputs W and Y form said third input;

15

wherein said sum generator and said carry-out generator are able to provide a sum bit and a carry-out bit, respectively, on said first, second, and third bits.

25. The apparatus of Claims 5, 7, 8, 9, 15, 17, or 24, wherein said adders are arranged in a Wallace tree.

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26. A machine method to obtain a product P from terms X and Y and accumulate it with a term Z, the result for use

in a digital processing system, wherein P is on of the group $P_{tc} = X_{tc} Y_{tc}$, $P_{uns} = X_{uns} Y_{uns}$, $P_A = X_{tc} Y_{uns}$, and $P_B = X_{uns} Y_{tc}$, wherein X is representable as vector $(a_{m-1}, a_{m-2}, \dots, a_0)$ and Y as vector $(b_{n-1}, b_{n-2}, \dots, b_0)$, comprising the steps of:

5 forming $L = a_{m-1} b_{n-1} 2^{m+n-2}$;

forming $M = \sum \sum a_i b_j 2^{i+j}$ for i from 0 to $m-2$ and j from 0 to $n-2$;

forming $N = \sum a_i b_{n-1} 2^{n-1+i}$ for i from 0 to $m-2$;

forming $O = \sum a_{m-1} b_j 2^{m-1+j}$ for j from 0 to $n-2$;

10 forming one or more compensations L_{2RC} , N_{2RC} , and O_{2RC} , in the event that one or more of L , N , and O , respectively, are negative;

forming P of one of the group P_{uns} , P_{tc} , P_A , and P_B , wherein P_{uns} is formed by summing L , M , N , and O , P_{tc} is formed by
 15 summing L , M , N_{2RC} , and O_{2RC} , P_A is formed by summing L_{2RC} , M , N , and O_{2RC} , and P_B is formed by summing L_{2RC} , M , N_{2RC} , and O ;
 and

accumulating P with Z .

27. A machine method to form a product P of $X=(a_{m-1}, a_{m-2}, \dots, a_0)$ and $Y=(b_{n-1}, b_{n-2}, \dots, b_0)$, and accumulate P with a term Z , wherein the result is for use in a digital processing system, and wherein each of X and Y may be in two's complement mode or in unsigned mode, comprising the steps of:

forming terms $a_i b_j$ for all i and j yielding positive values;

forming a combination of positive terms equivalent to any combination of terms $a_i b_j$ which is negative;

selectively combining said terms and said combination of positive terms to form P ; and

accumulating P with Z .

28. A machine method to form a product $P=(p_{m+n-1}, p_{m+n-2}, \dots, p_0)$ of $X=(a_{m-1}, a_{m-2}, \dots, a_0)$ and $Y=(b_{n-1}, b_{n-2}, \dots, b_0)$, and accumulate P with a term Z , wherein the result is for use in a digital processing system, and wherein each of X and Y may be in two's complement mode or in unsigned mode, comprising the steps of:

providing complement terms A_i and B_j , respectively, for $0 \leq i \leq m-1$ and $0 \leq j \leq n-1$;

41

providing terms a_{m-1} , b_{n-1} , a_i , b_j , a_i , b_{n-1} , A_i , b_{n-1} , a_{m-1} , b_j , and a_{m-1} , B_j , for $0 \leq i \leq m-2$ and $0 \leq j \leq n-2$;

providing a term which is the complement of said term a_{m-1} b_{n-1} ;

5 providing at least one term having the constant value 1; and

selectively combining said terms so formed, including:

forming p_k for $0 \leq k \leq m+n-2$, to form P for X and Y both in unsigned mode, by summing said terms a_i , b_j where $i+j=k$,
 10 and carries from the sum for $k-1$;

forming p_k for $0 \leq k \leq m+n-1$, to form P for X and Y both in two's complement mode,

by adding to the sum for each k for $0 \leq i \leq m-2$ and $0 \leq j \leq n-2$ said terms a_i , b_j , a_{m-1} , b_{n-1} , A_i , b_{n-1} , a_{m-1} , B_j ,
 15 and a_{m-1} , b_{n-1} , wherein the sum of the subscripts within each term is equal to k ,

by adding said terms a_{m-1} and b_{n-1} to the sum for $k=m+n-2$,

by adding said terms A_{m-1} , B_{n-1} , and 1 to the sum
 20 for $k=m+n-1$, and

by adding carries from the sum for each $k-1$ to the sum for each k ;

forming p_k for $0 \leq k \leq m+n-1$, to form P for X in two's complement mode and Y in unsigned mode,

5 by adding to the sum for each k for $0 \leq i \leq m-2$ and $0 \leq j \leq n-2$ said terms $a_i b_j$, $a_{m-1} a_i b_{n-1}$, $a_{m-1} B_j$, and $A_{m-1} B_{n-1}$, wherein the sum of the subscripts within each term is equal to k ,

10 by adding said terms a_{m-1} and 1 to the sum for $k=m+n-2$,

by adding said terms A_{m-1} and 1 to the sum for $k=m+n-1$, and

by adding carries from the sum for each $k-1$ to the sum for each k ;

15 forming p_k for $0 \leq k \leq m+n-1$, to form P for X in unsigned mode and Y in two's complement mode,

20 by adding to the sum for each k for $0 \leq i \leq m-2$ and $0 \leq j \leq n-2$ said terms $a_i b_j$, $b_{n-1} A_i b_{n-1}$, $a_{m-1} b_j$, and $A_{m-1} B_{n-1}$, wherein the sum of the subscripts within each term is equal to k ,

43

by adding said terms b_{n-1} and 1 to the sum for $k=m+n-2$,

by adding said terms B_{n-1} and 1 to the sum for $k=m+n-1$, and

5 by adding carries from the sum for each $k-1$ to the sum for each k ; and

accumulating P with Z .

29. A machine method to obtain a product P and accumulate it with a term Z , the result for use in a digital processing system, wherein P is one of the group $P_{tc}=X_{tc} Y_{tc}$, $P_{uns}=X_{uns} Y_{uns}$, $P_A=X_{tc} Y_{uns}$, and $P_B=X_{uns} Y_{tc}$, wherein X is representable as vector $(a_{m-1}, a_{m-2}, \dots, a_0)$ and Y as vector $(b_{n-1}, b_{n-2}, \dots, b_0)$, comprising the steps of:

10

forming

15
$$L = a_{m-1} b_{n-1} 2^{m+n-2},$$

$$M = \sum \sum a_i b_j 2^{i+j} \text{ for } i \text{ from } 0 \text{ to } m-2 \text{ and } j \text{ from } 0 \text{ to } n-2,$$

$$N = \sum a_i b_{n-1} 2^{n-1+i} \text{ for } i \text{ from } 0 \text{ to } m-2, \text{ and}$$

$$O = \sum a_{m-1} b_j 2^{m-1+j} \text{ for } j \text{ from } 0 \text{ to } n-2;$$

forming one or more compensations L_{2RC} , N_{2RC} , and O_{2RC} , in the event that one or more of L , N , and O , respectively, are negative; and

forming P of one of the group P_{uns} , P_{tc} , P_A , and P_B , wherein
5 P_{uns} is formed by summing L , M , N , and O , P_{tc} is formed by summing L , M , N_{2RC} , and O_{2RC} , P_A is formed by summing L_{2RC} , M , N , and O_{2RC} , and P_B is formed by summing L_{2RC} , M , N_{2RC} , and O ;

selecting one of the group P_{uns} , P_{tc} , P_A , and P_B , to provide as P ; and

10 accumulating P with Z .

$$X = \sum_{i=0}^{m-1} a_i 2^i$$

FIG. 1a

$$Y = \sum_{j=0}^{n-1} b_j 2^j$$

FIG. 1b

$$Z = \sum_{i=0}^{m+n-1} c_i 2^i$$

FIG. 1c

$$X_{tc} = -a_{m-1} 2^{m-1} + \sum_{i=0}^{m-2} a_i 2^i \quad \text{FIG. 2a}$$

$$Y_{tc} = -b_{n-1} 2^{n-1} + \sum_{j=0}^{n-2} b_j 2^j \quad \text{FIG. 2b}$$

$$Z_{tc} = -c_{m+n-1} 2^{m+n-1} + \sum_{i=0}^{m+n-2} c_i 2^i \quad \text{FIG. 2c}$$

$$X_{uns} = a_{m-1} 2^{m-1} + \sum_{i=0}^{m-2} a_i 2^i \quad \text{FIG. 2d}$$

$$Y_{uns} = b_{n-1} 2^{n-1} + \sum_{j=0}^{n-2} b_j 2^j \quad \text{FIG. 2e}$$

$$Z_{uns} = c_{m+n-1} 2^{m+n-1} + \sum_{i=0}^{m+n-2} c_i 2^i \quad \text{FIG. 2f}$$

$$P_{tc} = \underbrace{+a_{m-1} b_{n-1} 2^{m+n-2}}_L + \underbrace{\sum_{i=0}^{m-2} \sum_{j=0}^{m-2} a_i b_j 2^{i+j}}_M - \underbrace{\sum_{i=0}^{m-2} a_i b_{n-1} 2^{n-1+i}}_N - \underbrace{\sum_{j=0}^{n-2} a_{m-1} b_j 2^{m-1+j}}_O$$

FIG. 3a

$$P_{uns} = +a_{m-1} b_{n-1} 2^{m+n-2} + \sum_{i=0}^{m-2} \sum_{j=0}^{m-2} a_i b_j 2^{i+j} + \sum_{i=0}^{m-2} a_i b_{n-1} 2^{n-1+i} + \sum_{j=0}^{n-2} a_{m-1} b_j 2^{m-1+j}$$

FIG. 3b

$$P_A = -a_{m-1} b_{n-1} 2^{m+n-2} + \sum_{i=0}^{m-2} \sum_{j=0}^{m-2} a_i b_j 2^{i+j} + \sum_{i=0}^{m-2} a_i b_{n-1} 2^{n-1+i} - \sum_{j=0}^{n-2} a_{m-1} b_j 2^{m-1+j}$$

FIG. 3c

$$P_B = -a_{m-1} b_{n-1} 2^{m+n-2} + \sum_{i=0}^{m-2} \sum_{j=0}^{n-2} a_i b_j 2^{i+j} - \sum_{i=0}^{m-2} a_i b_{n-1} 2^{n-1+i} + \sum_{j=0}^{n-2} a_{m-1} b_j 2^{m-1+j}$$

FIG. 3d

$$L = a_{m-1}b_{n-1}2^{m+n-2}$$

$$\left\{ \begin{array}{l} 0 \cdot 2^{m+n} + 0 \cdot 2^{m+n-1} + a_{m-1}b_{n-1}2^{m+n-2} + 0 \cdot 2^{m+n-3} + \dots + 0 \cdot 2^0 \end{array} \right\}$$

$$\left\{ \begin{array}{l} 1 \cdot 2^{m+n} + 1 \cdot 2^{m+n-1} + \overline{a_{m-1}b_{n-1}}2^{m+n-2} + 1 \cdot 2^{m+n-3} + \dots + 1 \cdot 2^0 + 1 \end{array} \right\}$$

$$\left\{ \begin{array}{l} 1 \cdot 2^{m+n} + 1 \cdot 2^{m+n-1} + \overline{a_{m-1}b_{n-1}}2^{m+n-2} + 1 \cdot 2^{m+n-3} + \dots + 0 \cdot 2^0 \end{array} \right\}$$

FIG. 4a

FIG. 4b

FIG. 4c

FIG. 4d

FIG. 4e

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{array}{ccc} 0 & 0 & 0 \\ 1 & 1 & \dots \\ \overline{a_{m-1}b_{n-1}} & 0 & 0 \end{array}$$

4/36

$$N = \sum_{i=0}^{m-2} a_i b_{n-1} 2^{n-1+i}$$

FIG. 5a

$$0 \cdot 2^{m+n} + 0 \cdot 2^{m+n-1} + 0 \cdot 2^{m+n-2} + \sum_{i=0}^{m-2} a_i b_{n-1} 2^{n-1+i}$$

FIG. 5b

$$0 \cdot 2^{m+n} + 0 \cdot 2^{m+n-1} + 0 \cdot 2^{m+n-2} + \sum_{i=0}^{m-2} a_i 2^{n-1+i}$$

FIG. 5c

$$1 \cdot 2^{m+n} + 1 \cdot 2^{m+n-1} + 1 \cdot 2^{m+n-2} + 1 + \sum_{i=0}^{m-2} \bar{a}_i 2^{n-1+i}$$

FIG. 5d

$$1 \cdot 2^{m+n} + 1 \cdot 2^{m+n-1} + \bar{b}_{n-1} 2^{m+n-1} + b_{n-1} 2^{m+n-2} + b_{n-1} + \sum_{i=0}^{m-2} \bar{a}_i b_{n-1} 2^{n-1+i}$$

FIG. 5e

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} \bar{b}_{n-1} & b_{n-1} & \bar{a}_{m-2} b_{n-1} & \bar{a}_{m-3} b_{n-1} & \dots & \bar{a}_1 b_{n-1} & \bar{a}_0 b_{n-1} \\ 1 & 0 & 0 & 0 & \dots & 0 & b_{n-1} \end{bmatrix}$$

FIG. 5f

$$O = \sum_{j=0}^{n-2} a_{m-1} b_j 2^{m-1+j} \quad \text{FIG. 6a}$$

$$0 \cdot 2^{m+n} + 0 \cdot 2^{m+n-1} + 0 \cdot 2^{m+n-2} + \sum_{j=0}^{n-2} a_{m-1} b_j 2^{m-1+j} \quad \text{FIG. 6b}$$

$$0 \cdot 2^{m+n} + 0 \cdot 2^{m+n-1} + 0 \cdot 2^{m+n-2} + \sum_{j=0}^{n-2} b_j 2^{m-1+j} \quad \text{FIG. 6c}$$

$$1 \cdot 2^{m+n} + 1 \cdot 2^{m+n-1} + 1 \cdot 2^{m+n-2} + 1 + \sum_{j=0}^{n-2} \overline{b_j} 2^{m-1+j} \quad \text{FIG. 6d}$$

$$1 \cdot 2^{m+n} + 1 \cdot 2^{m+n-1} + \overline{a_{m-1}} 2^{m+n-1} + a_{m-1} 2^{m+n-2} + a_{m-1} + \sum_{j=0}^{n-2} a_{m-1} \overline{b_j} 2^{m-1+j} \quad \text{FIG. 6e}$$

$$\begin{bmatrix} 0 & \overline{a_{m-1}} & a_{m-1} & \overline{a_{m-1} b_{n-2}} & \overline{a_{m-1} b_{n-3}} & \dots & a_{m-1} \overline{b_1} & a_{m-1} \overline{b_0} \\ 1 & 1 & 0 & 0 & 0 & \dots & 0 & a_{m-1} \end{bmatrix} \quad \text{FIG. 6f}$$

6/36

$$Z_{lc} = -c_{m+n-1} 2^{m+n-1} + \sum_{i=0}^{m+n-2} c_i 2^i \quad \text{FIG. 7a}$$

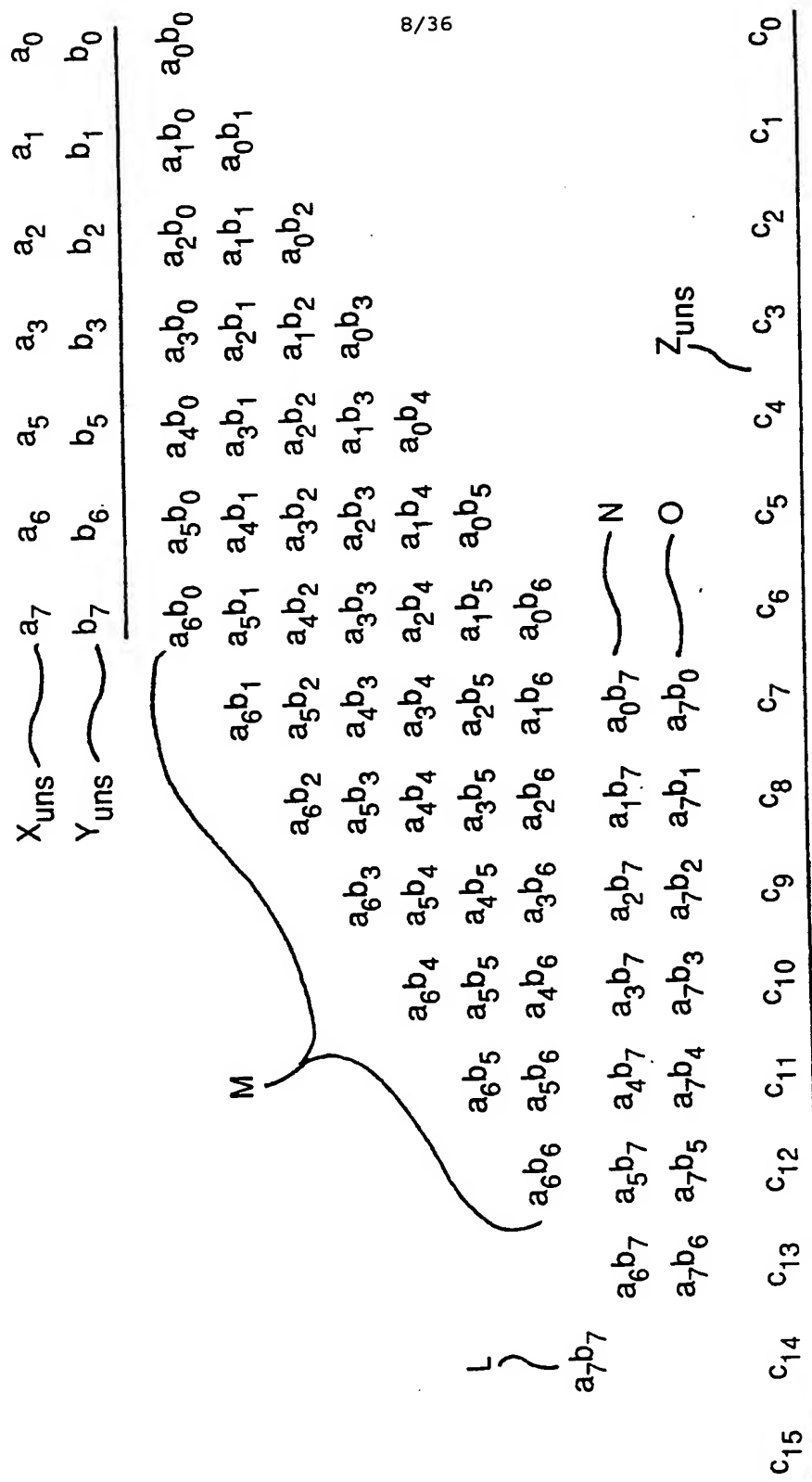
$$c_{m+n-1} 2^{m+n-1} = 0 \cdot 2^{m+n} + c_{m+n-1} 2^{m+n-1} + 0 \cdot 2^{m+n-2} + \dots + 0 \cdot 2^0 \quad \text{FIG. 7b}$$

$$-c_{m+n-1} 2^{m+n-1} = 1 \cdot 2^{m+n} + \overline{c_{m+n-1}} 2^{m+n-1} + 1 \cdot 2^{m+n-2} + \dots + 1 \cdot 2^0 + 1 \quad \text{FIG. 7c}$$

$$-c_{m+n-1} 2^{m+n-1} = 1 \cdot 2^{m+n} + \overline{c_{m+n-1}} 2^{m+n-1} + 1 \cdot 2^{m+n-1} + 0 \cdot 2^{m+n-2} + \dots + 0 \cdot 2^0 \quad \text{FIG. 7d}$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} \overline{c_{m+n-1}} & 0 & \dots & 0 \\ 1 & 0 & \dots & 0 \end{bmatrix} \quad \text{FIG. 7e}$$

FIG. 8a



8/36

9/36

	a_7	a_6	a_5	a_3	a_2	a_1	a_0
	b_7	b_6	b_5	b_3	b_2	b_1	b_0
	a_7b_0	a_6b_0	a_5b_0	a_4b_0	a_3b_0	a_2b_0	a_1b_0
		a_6b_1	a_5b_1	a_4b_1	a_3b_1	a_2b_1	a_1b_1
		a_6b_2	a_5b_2	a_4b_2	a_3b_2	a_2b_2	a_1b_2
		a_6b_3	a_5b_3	a_4b_3	a_3b_3	a_2b_3	a_1b_3
		a_6b_4	a_5b_4	a_4b_4	a_3b_4	a_2b_4	a_1b_4
		a_6b_5	a_5b_5	a_4b_5	a_3b_5	a_2b_5	a_1b_5
		a_6b_6	a_5b_6	a_4b_6	a_3b_6	a_2b_6	a_1b_6
		a_6b_7	a_5b_7	a_4b_7	a_3b_7	a_2b_7	a_1b_7
a_7b_7							
c_{15}	c_{14}	c_{13}	c_{12}	c_{11}	c_{10}	c_9	c_8
						c_7	c_6
						c_5	c_4
						c_3	c_2
						c_1	c_0

FIG. 8c

10/36

c_7 a_7b_0 a_6b_1	c_6 a_6b_0 a_5b_1	c_5 a_5b_0 a_4b_1	c_4 a_4b_0 a_3b_1	c_3 a_3b_0 a_2b_1	c_2 a_2b_0 a_1b_1	c_1 a_1b_0 a_0b_1	c_0 a_0b_0
c_8 a_7b_1 a_6b_2	a_5b_2	a_4b_2	a_3b_2	a_2b_2	a_1b_2	a_0b_2	
c_9 a_7b_2 a_6b_3	a_5b_3	a_4b_3	a_3b_3	a_2b_3	a_1b_3	a_0b_3	
c_{10} a_7b_3 a_6b_4	a_5b_4	a_4b_4	a_3b_4	a_2b_4	a_1b_4	a_0b_4	
c_{11} a_7b_4 a_6b_5	a_5b_5	a_4b_5	a_3b_5	a_2b_5	a_1b_5	a_0b_5	
c_{12} a_7b_5 a_6b_6	a_5b_6	a_4b_6	a_3b_6	a_2b_6	a_1b_6	a_0b_6	
c_{13} a_7b_6 a_6b_7	a_5b_7	a_4b_7	a_3b_7	a_2b_7	a_1b_7	a_0b_7	
c_{14}							

 c_{15}

FIG. 9a

FIG. 9a

11/36

$$A = \bar{a} \quad B = \bar{b} \quad C = \bar{c}$$

		X_{1c}	a_7	a_6	a_5	a_3	a_2	a_1	a_0	
		Y_{1c}	b_7	b_6	b_5	b_3	b_2	b_1	b_0	
L	}	$a_7 b_7$	M	$a_6 b_0$	$a_5 b_0$	$a_4 b_0$	$a_3 b_0$	$a_2 b_0$	$a_1 b_0$	$a_0 b_0$
				$a_6 b_1$	$a_5 b_1$	$a_4 b_1$	$a_3 b_1$	$a_2 b_1$	$a_1 b_1$	$a_0 b_1$
				$a_6 b_2$	$a_5 b_2$	$a_4 b_2$	$a_3 b_2$	$a_2 b_2$	$a_1 b_2$	$a_0 b_2$
				$a_6 b_3$	$a_5 b_3$	$a_4 b_3$	$a_3 b_3$	$a_2 b_3$	$a_1 b_3$	$a_0 b_3$
				$a_6 b_4$	$a_5 b_4$	$a_4 b_4$	$a_3 b_4$	$a_2 b_4$	$a_1 b_4$	$a_0 b_4$
				$a_6 b_5$	$a_5 b_5$	$a_4 b_5$	$a_3 b_5$	$a_2 b_5$	$a_1 b_5$	$a_0 b_5$
				$a_6 b_6$	$a_5 b_6$	$a_4 b_6$	$a_3 b_6$	$a_2 b_6$	$a_1 b_6$	$a_0 b_6$
B_7	1	0	0	0	0	0	0	0	0	0
A_7	1	0	0	0	0	0	0	0	0	0
C_{15}	1	0	0	0	0	0	0	0	0	0

N_{2RC}
 O_{2RC}
 Z_{2RC}

$$A=\overline{a} \quad B=\overline{b} \quad C=\overline{c}$$

	a_7	a_6	a_5	a_3	a_2	a_1	a_0
	b_7	b_6	b_5	b_3	b_2	b_1	b_0
	a_7B_0	a_6b_0	a_5b_0	a_4b_0	a_3b_0	a_2b_0	a_1b_0
		a_6b_1	a_5b_1	a_4b_1	a_3b_1	a_2b_1	a_1b_1
	a_7B_1	a_6b_2	a_5b_2	a_4b_2	a_3b_2	a_2b_2	a_1b_2
		a_6b_3	a_5b_3	a_4b_3	a_3b_3	a_2b_3	a_1b_3
	a_7B_2	a_6b_4	a_5b_4	a_4b_4	a_3b_4	a_2b_4	a_1b_4
		a_6b_5	a_5b_5	a_4b_5	a_3b_5	a_2b_5	a_1b_5
	a_7B_3	a_6b_6	a_5b_6	a_4b_6	a_3b_6	a_2b_6	a_1b_6
		a_6b_7	a_5b_7	a_4b_7	a_3b_7	a_2b_7	a_1b_7
	a_7B_4	a_6b_0	a_5b_0	a_4b_0	a_3b_0	a_2b_0	a_1b_0
		a_6b_1	a_5b_1	a_4b_1	a_3b_1	a_2b_1	a_1b_1
	a_7B_5	a_6b_2	a_5b_2	a_4b_2	a_3b_2	a_2b_2	a_1b_2
		a_6b_3	a_5b_3	a_4b_3	a_3b_3	a_2b_3	a_1b_3
	a_7B_6	a_6b_4	a_5b_4	a_4b_4	a_3b_4	a_2b_4	a_1b_4
		a_6b_5	a_5b_5	a_4b_5	a_3b_5	a_2b_5	a_1b_5
	a_7B_7	a_6b_6	a_5b_6	a_4b_6	a_3b_6	a_2b_6	a_1b_6
		a_6b_7	a_5b_7	a_4b_7	a_3b_7	a_2b_7	a_1b_7
1	a_7b_7						
B_7	b_7						
A_7	a_7						

FIG. 9c

13/36

$$A = \bar{a} \quad B = \bar{b} \quad C = \bar{c}$$

		c_7 a_7b_0 a_6b_1	c_6 a_6b_0 a_5b_1	c_5 a_5b_0 a_4b_1	c_4 a_4b_0 a_3b_1	c_3 a_3b_0 a_2b_1	c_2 a_2b_0 a_1b_1	c_1 a_1b_0 a_0b_1	c_0 a_0b_0
		c_8 a_7b_1 a_6b_2	a_5b_2 a_7	a_4b_2	a_3b_2	a_2b_2	a_1b_2	a_0b_2	
		c_9 a_7b_2 a_6b_3	a_5b_3	a_4b_3	a_3b_3	a_2b_3	a_1b_3	a_0b_3	
		c_{10} a_7b_3 a_6b_4	a_5b_4	a_4b_4	a_3b_4	a_2b_4	a_1b_4	a_0b_4	
		c_{11} a_7b_4 a_6b_5	a_5b_5	a_4b_5	a_3b_5	a_2b_5	a_1b_5	a_0b_5	
		c_{12} a_7b_5 a_6b_6	a_5b_6	a_4b_6	a_3b_6	a_2b_6	a_1b_6	a_0b_6	
C_{15} A_7 B_7 1	a_7b_7 a_7 b_7 c_{14}	c_{13} a_7b_6 A_6b_7	A_5b_7	A_4b_7	A_3b_7	A_2b_7	A_1b_7	A_0b_7 b_7	

FIG. 10a

		A= \bar{a} B= \bar{b} C= \bar{c}													
		X_{lc} $\overbrace{a_7 \dots a_0}$							Y_{uns} $\overbrace{b_7 \dots b_0}$						
0	A_7B_7	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A_7	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C_{15}	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0

14/36

FIG. 10b

A= \overline{a} B= \overline{b} C= \overline{c}														
	a ₇	a ₆	a ₅	a ₃	a ₂	a ₁	a ₀							
	b ₇	b ₆	b ₅	b ₃	b ₂	b ₁	b ₀							
		a ₇ B ₀	a ₆ b ₀	a ₅ b ₀	a ₄ b ₀	a ₃ b ₀	a ₂ b ₀	a ₁ b ₀	a ₀ b ₀					
		a ₇ B ₁	a ₆ b ₁	a ₅ b ₁	a ₄ b ₁	a ₃ b ₁	a ₂ b ₁	a ₁ b ₁	a ₀ b ₁					
		a ₇ B ₂	a ₆ b ₂	a ₅ b ₂	a ₄ b ₂	a ₃ b ₂	a ₂ b ₂	a ₁ b ₂	a ₀ b ₂					
		a ₇ B ₃	a ₆ b ₃	a ₅ b ₃	a ₄ b ₃	a ₃ b ₃	a ₂ b ₃	a ₁ b ₃	a ₀ b ₃					
		a ₇ B ₄	a ₆ b ₄	a ₅ b ₄	a ₄ b ₄	a ₃ b ₄	a ₂ b ₄	a ₁ b ₄	a ₀ b ₄					
		a ₇ B ₅	a ₆ b ₅	a ₅ b ₅	a ₄ b ₅	a ₃ b ₅	a ₂ b ₅	a ₁ b ₅	a ₀ b ₅					
		a ₇ B ₆	a ₆ b ₆	a ₅ b ₆	a ₄ b ₆	a ₃ b ₆	a ₂ b ₆	a ₁ b ₆	a ₀ b ₆					
		a ₆ b ₇	a ₅ b ₇	a ₄ b ₇	a ₃ b ₇	a ₂ b ₇	a ₁ b ₇	a ₀ b ₇						
A ₇	a ₇													
C ₁₅	C ₁₄	C ₁₃	C ₁₂	C ₁₁	C ₁₀	C ₉	C ₈	C ₇	C ₆	C ₅	C ₄	C ₃	C ₂	C ₁ C ₀

15/36

FIG. 10c

16/36

$$A = \bar{a} \quad B = \bar{b} \quad C = \bar{c}$$

		c_7	c_6	c_5	c_4	c_3	c_2	c_1	c_0
		a_7B_0	a_6b_0	a_5b_0	a_4b_0	a_3b_0	a_2b_0	a_1b_0	a_0b_0
		a_6b_1	a_5b_1	a_4b_1	a_3b_1	a_2b_1	a_1b_1	a_0b_1	
		c_8							
		a_7B_1	a_5b_2	a_4b_2	a_3b_2	a_2b_2	a_1b_2	a_0b_2	
		a_6b_2	a_7						
		c_9							
		a_7B_2	a_5b_3	a_4b_3	a_3b_3	a_2b_3	a_1b_3	a_0b_3	
		a_6b_3							
		c_{10}							
		a_7B_3	a_5b_4	a_4b_4	a_3b_4	a_2b_4	a_1b_4	a_0b_4	
		a_6b_4							
		c_{11}							
		a_7B_4	a_5b_5	a_4b_5	a_3b_5	a_2b_5	a_1b_5	a_0b_5	
		a_6b_5							
		c_{12}							
		a_7B_5	a_5b_6	a_4b_6	a_3b_6	a_2b_6	a_1b_6	a_0b_6	
		a_6b_6							
		c_{13}							
		a_7B_6	a_5b_7	a_4b_7	a_3b_7	a_2b_7	a_1b_7	a_0b_7	
		a_6b_7							
C_{15}	A_7B_7								
A_7	a_7								
1	c_{14}								
	1								

SUBSTITUTE SHEET

FIG. 11c

19/36

$$A = \bar{a} \quad B = \bar{b} \quad C = \bar{c}$$

<div> <div> C_{15} B_7 1 </div> <div> $A_7 B_7$ b_7 c_{14} 1 </div> </div>	c_7 $a_7 b_0$ $a_6 b_1$	c_6 $a_6 b_0$ $a_5 b_1$	c_5 $a_5 b_0$ $a_4 b_1$	c_4 $a_4 b_0$ $a_3 b_1$	c_3 $a_3 b_0$ $a_2 b_1$	c_2 $a_2 b_0$ $a_1 b_1$	c_1 $a_1 b_0$ $a_0 b_1$	c_0 $a_0 b_0$
	c_8 $a_7 b_1$ $a_6 b_2$	$a_5 b_2$	$a_4 b_2$	$a_3 b_2$	$a_2 b_2$	$a_1 b_2$	$a_0 b_2$	
	c_9 $a_7 b_2$ $a_6 b_3$	$a_5 b_3$	$a_4 b_3$	$a_3 b_3$	$a_2 b_3$	$a_1 b_3$	$a_0 b_3$	
	c_{10} $a_7 b_3$ $a_6 b_4$	$a_5 b_4$	$a_4 b_4$	$a_3 b_4$	$a_2 b_4$	$a_1 b_4$	$a_0 b_4$	
	c_{11} $a_7 b_4$ $a_6 b_5$	$a_5 b_5$	$a_4 b_5$	$a_3 b_5$	$a_2 b_5$	$a_1 b_5$	$a_0 b_5$	
	c_{12} $a_7 b_5$ $a_6 b_6$	$a_5 b_6$	$a_4 b_6$	$a_3 b_6$	$a_2 b_6$	$a_1 b_6$	$a_0 b_6$	
	c_{13} $a_7 b_6$ $A_6 b_7$	$A_5 b_7$	$A_4 b_7$	$A_3 b_7$	$A_2 b_7$	$A_1 b_7$	$A_0 b_7$ b_7	

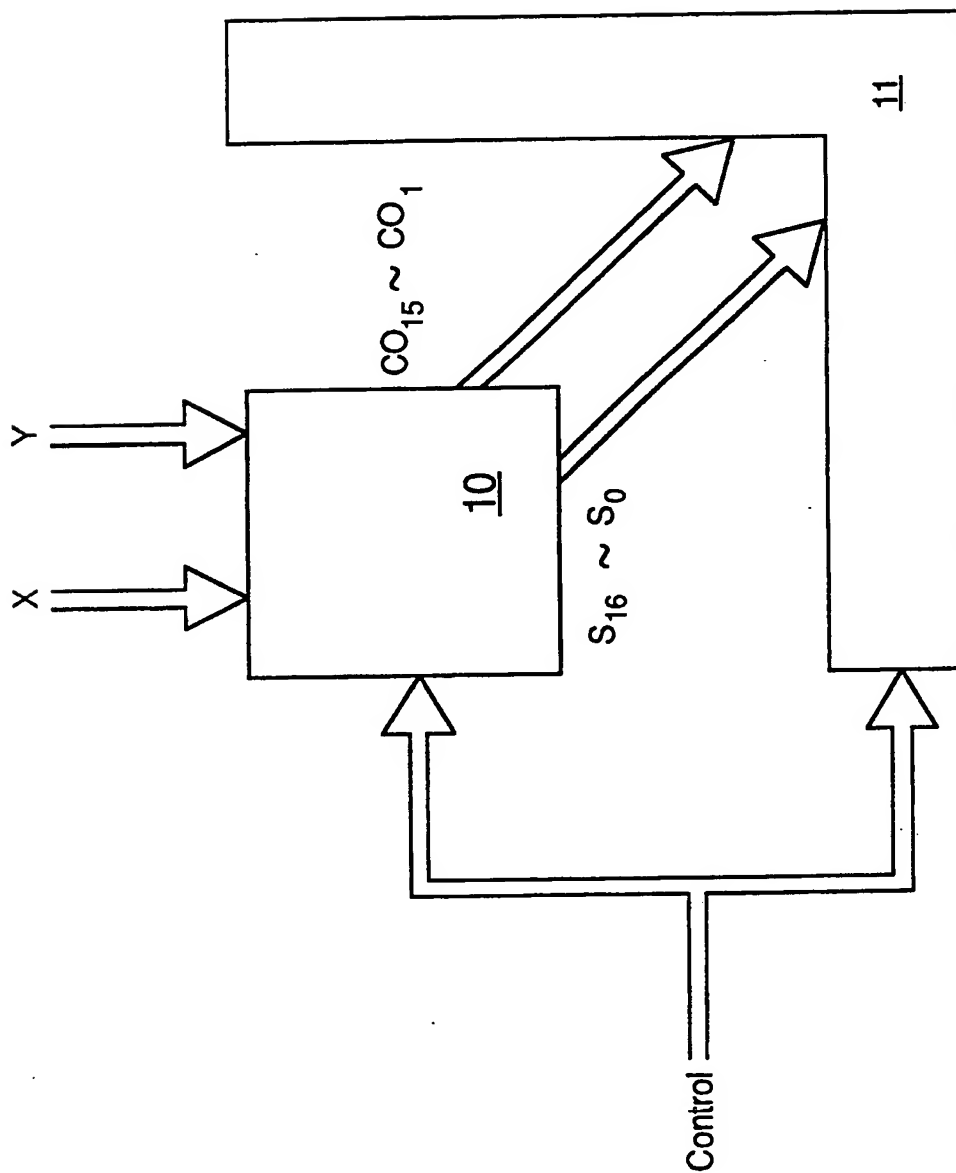
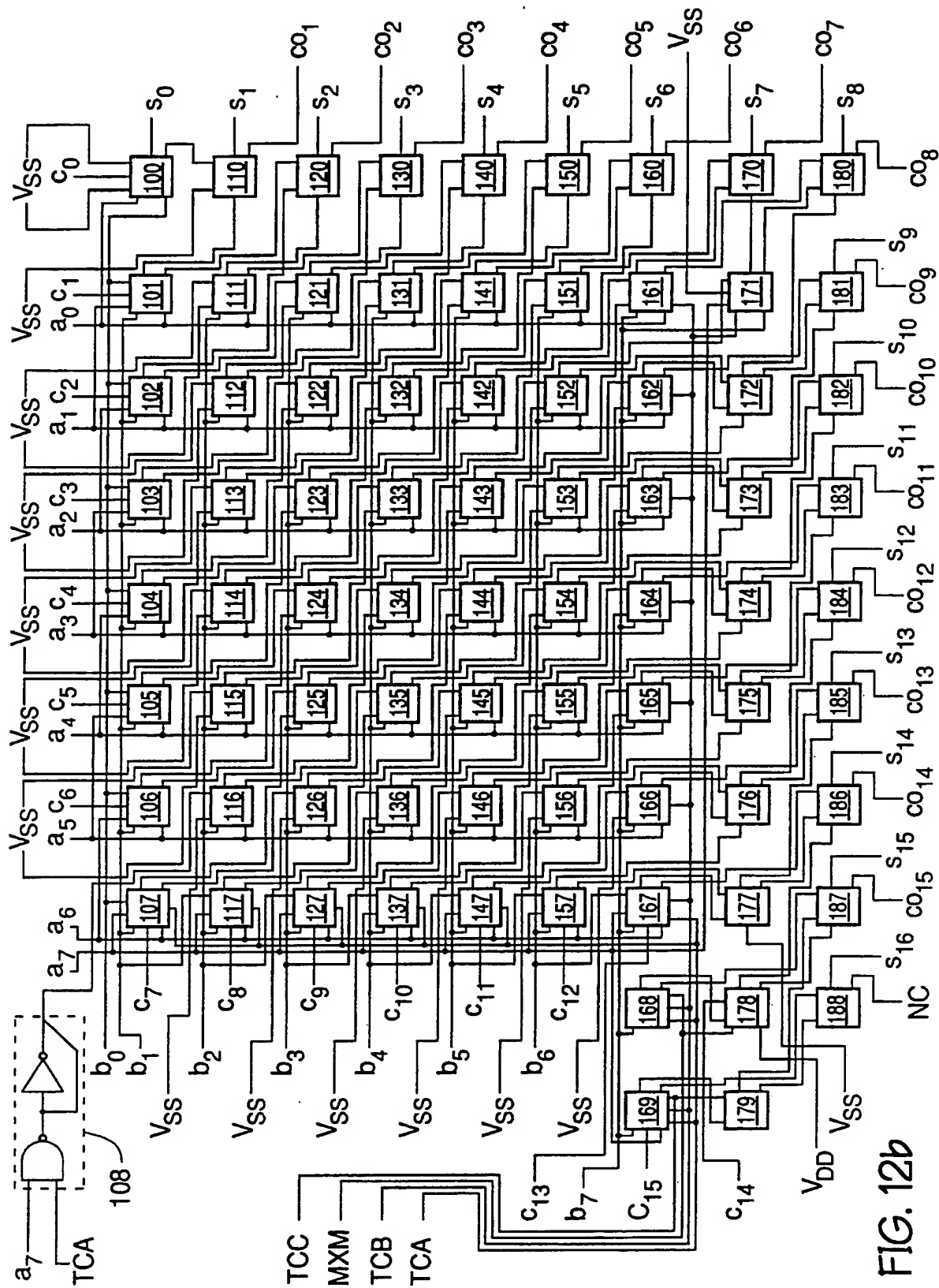


FIG. 12a



22/36

FIG. 13a

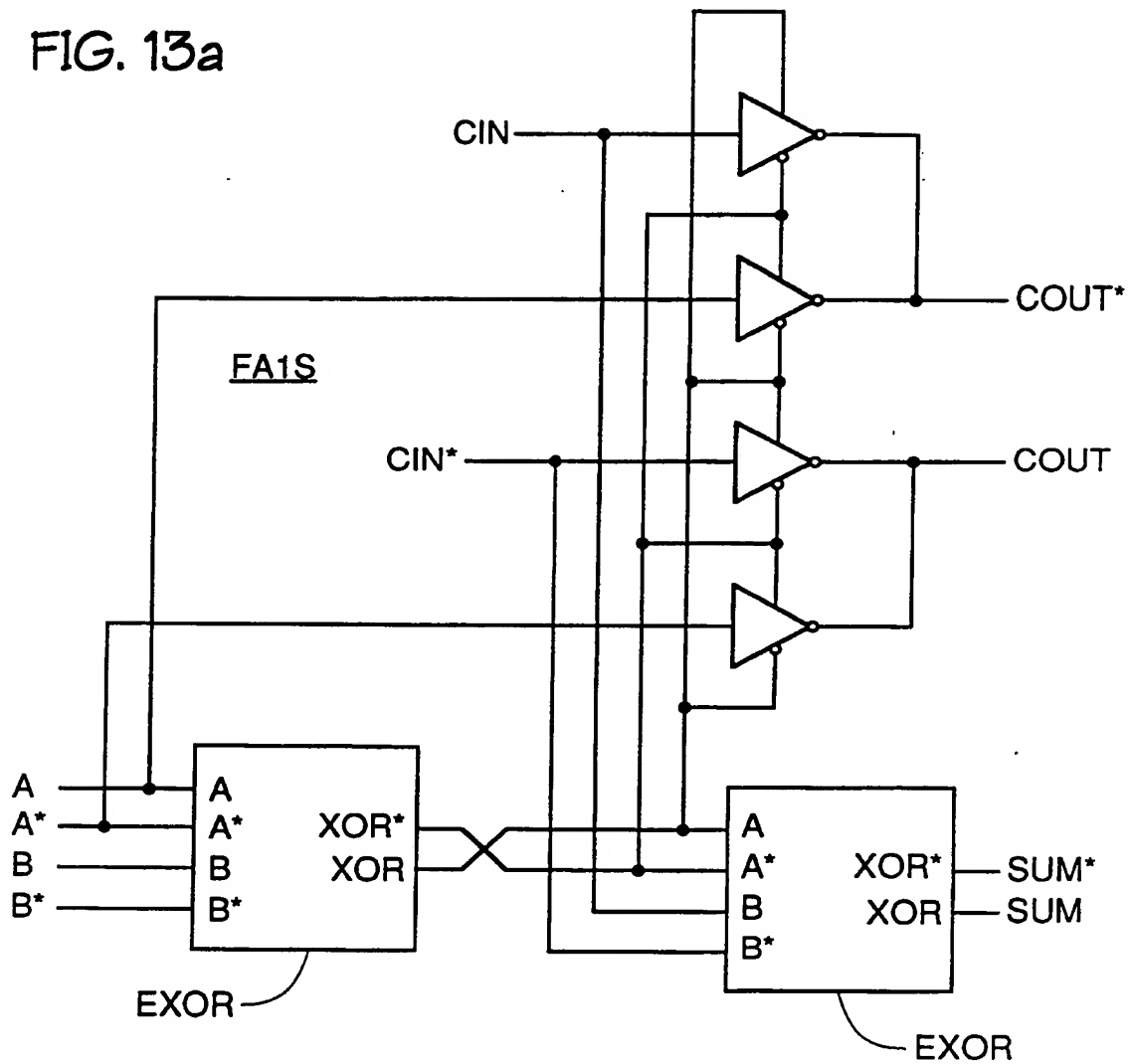
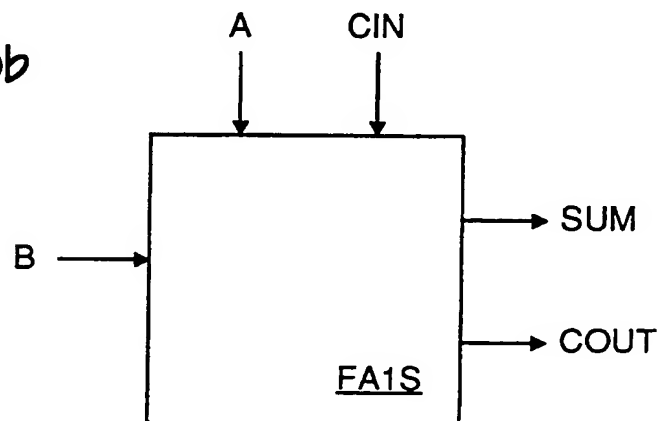


FIG. 13b



23/36

FIG. 14a

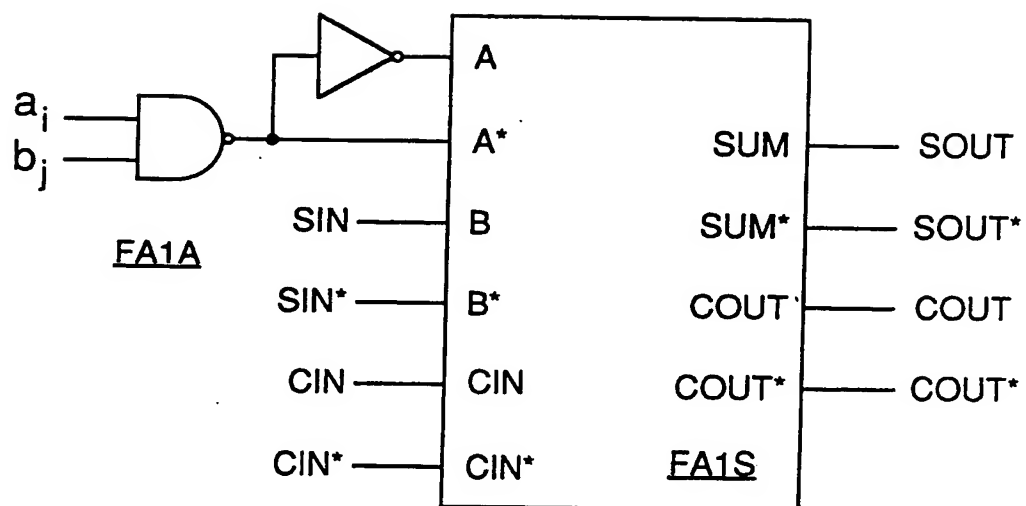


FIG. 14b

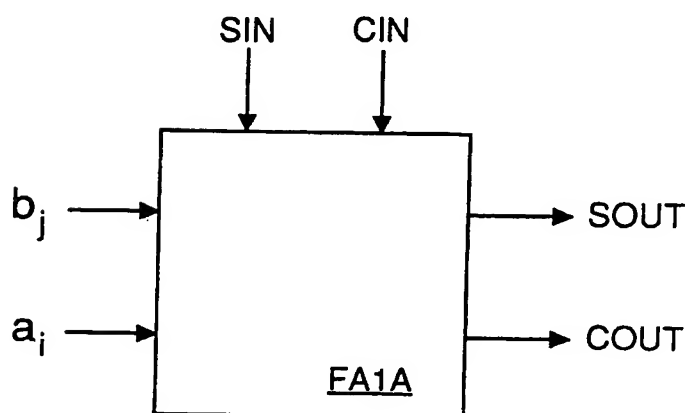


FIG. 15a

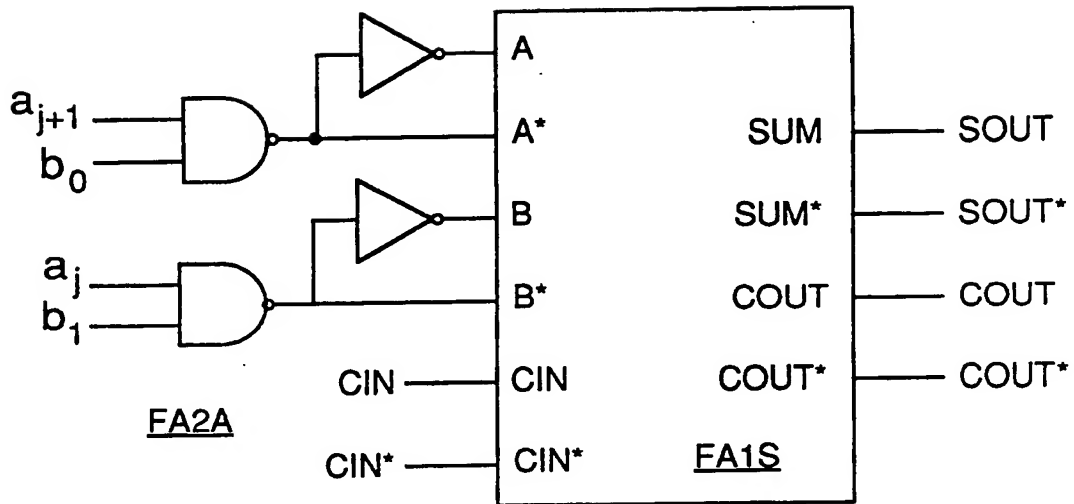


FIG. 15b

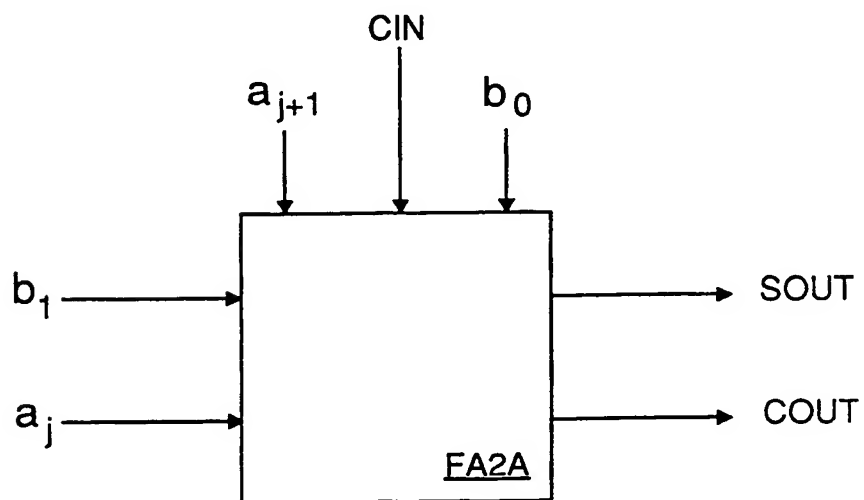


FIG. 16a

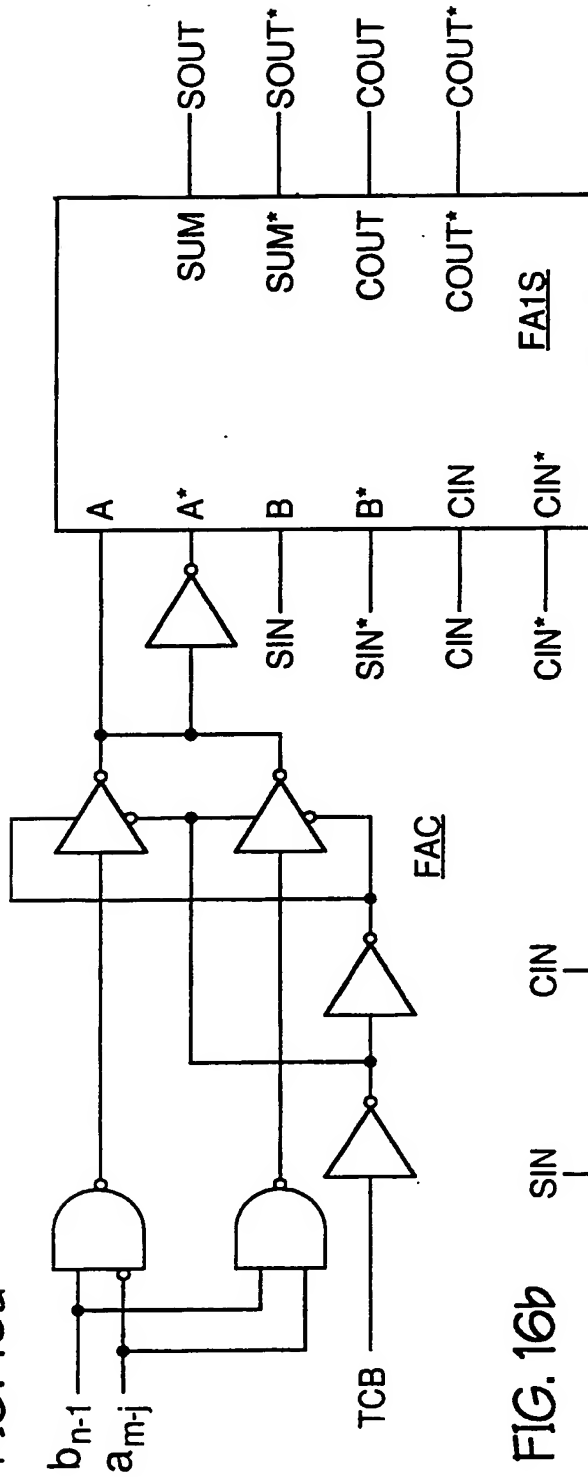


FIG. 16b

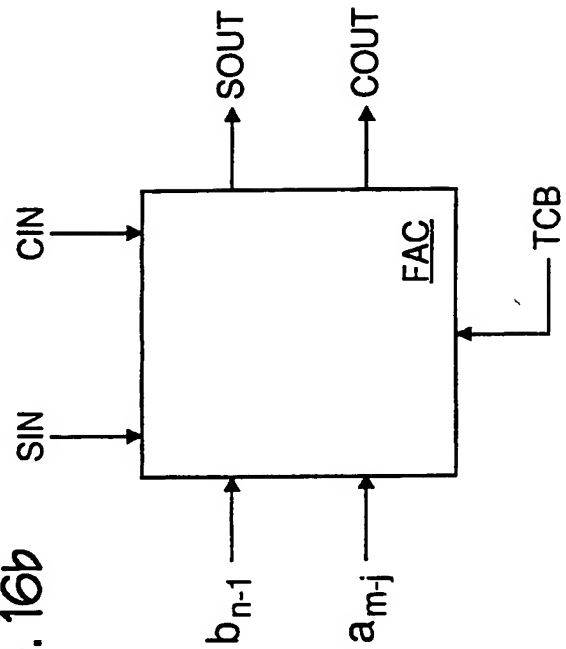


FIG. 17a

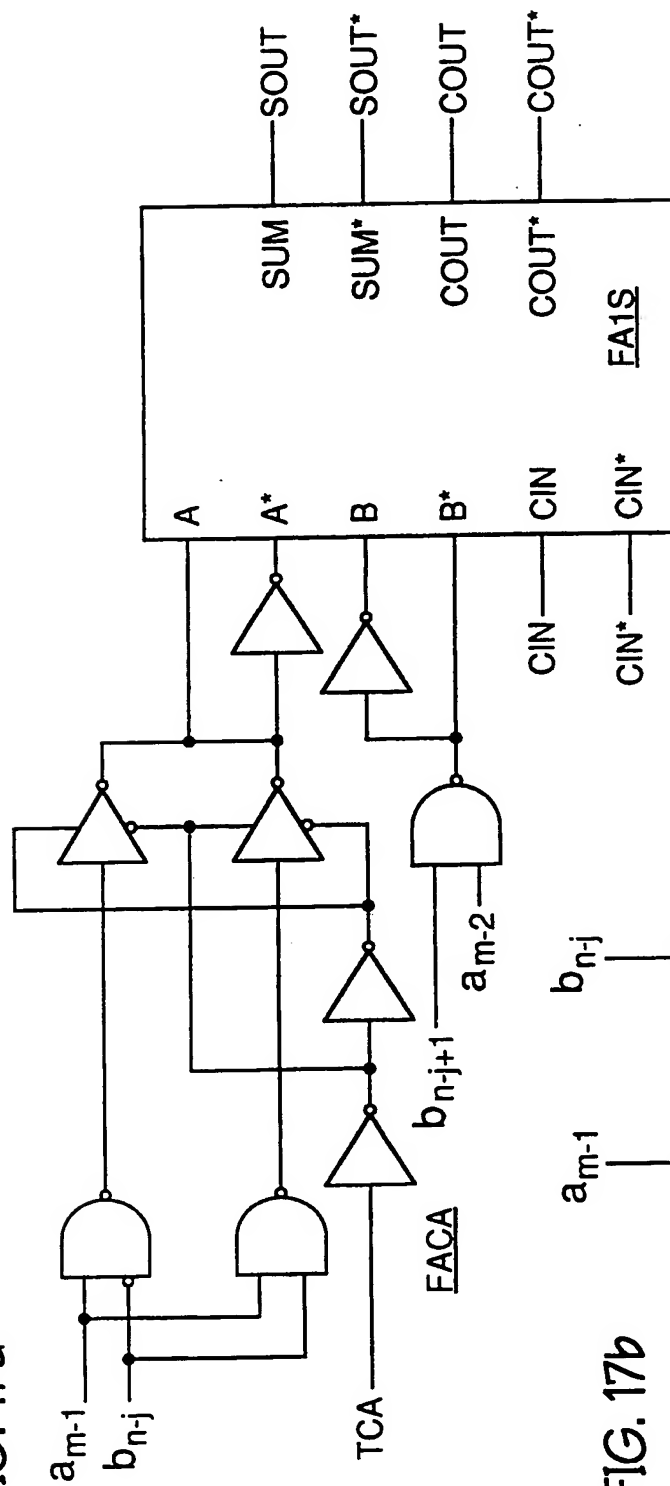
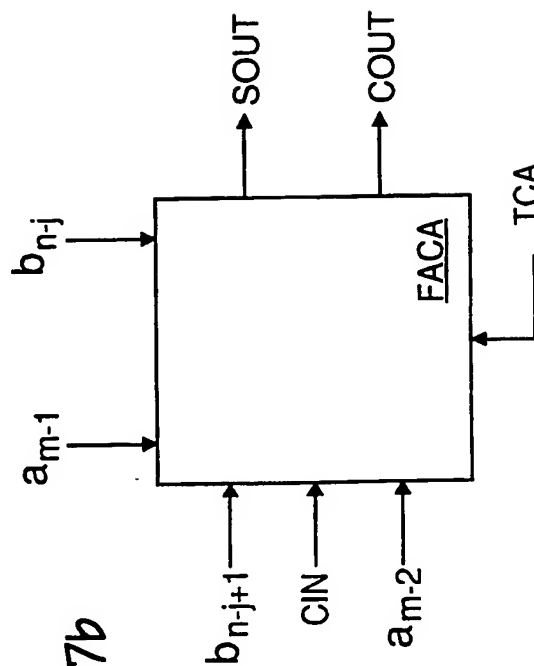


FIG. 17b



27/36

FIG. 18a

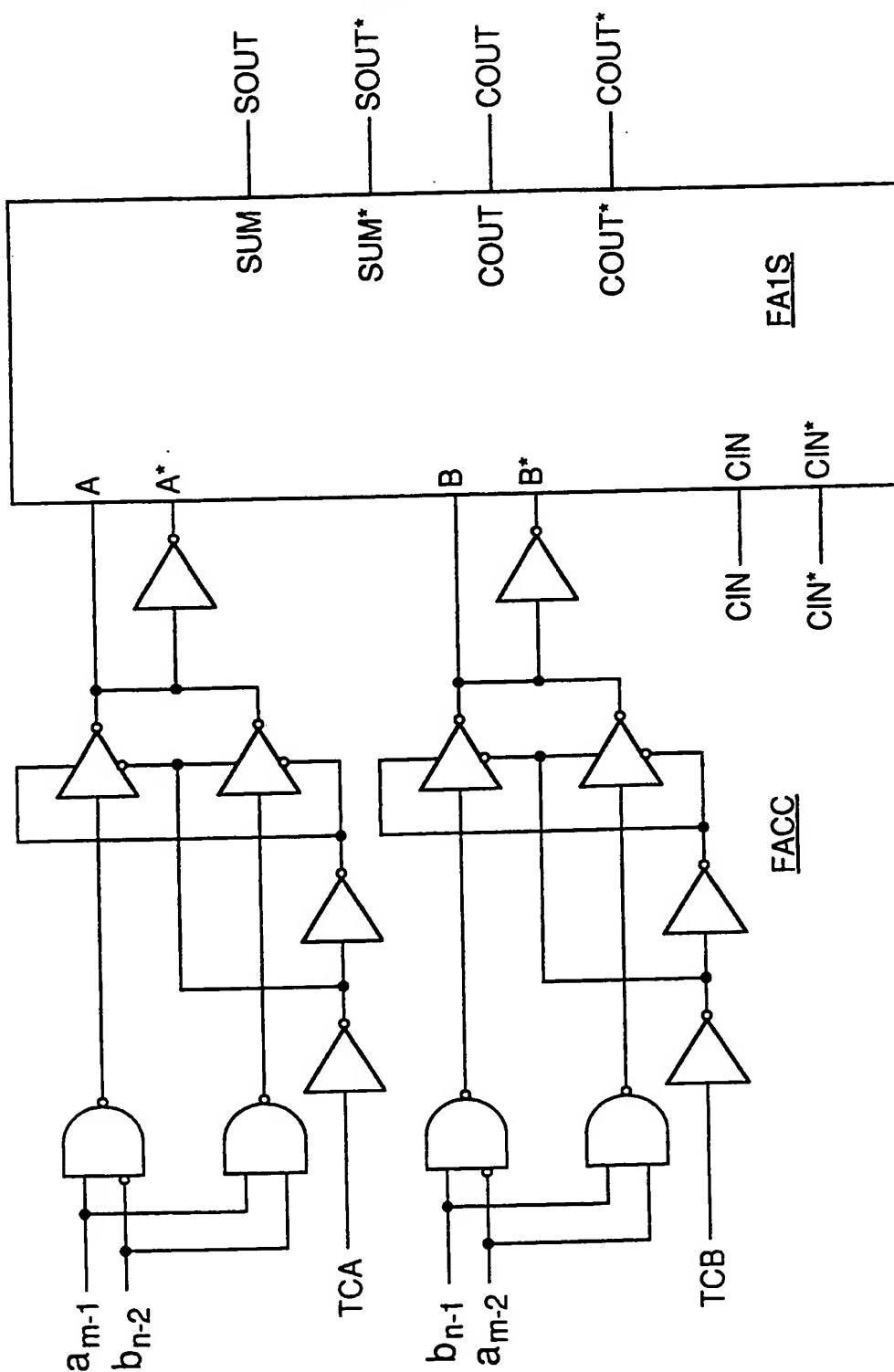
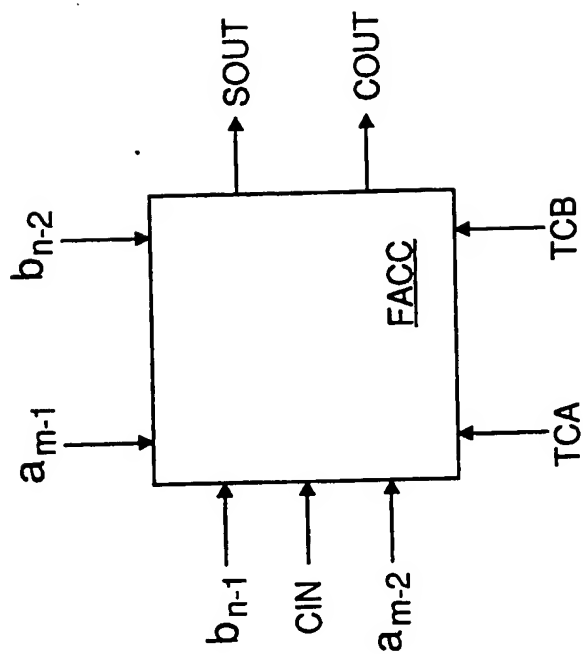


FIG. 18b



29/36

FIG. 19a

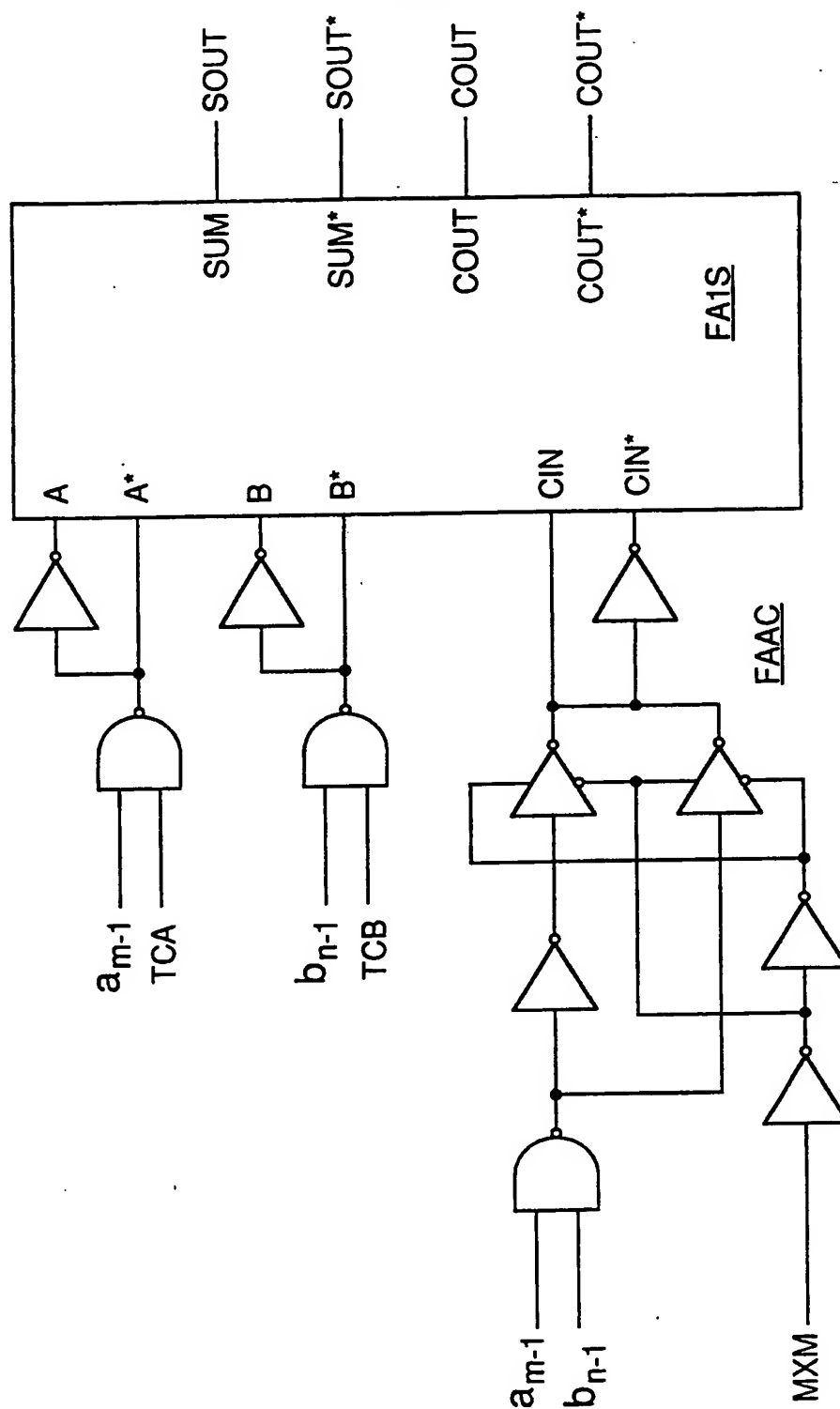
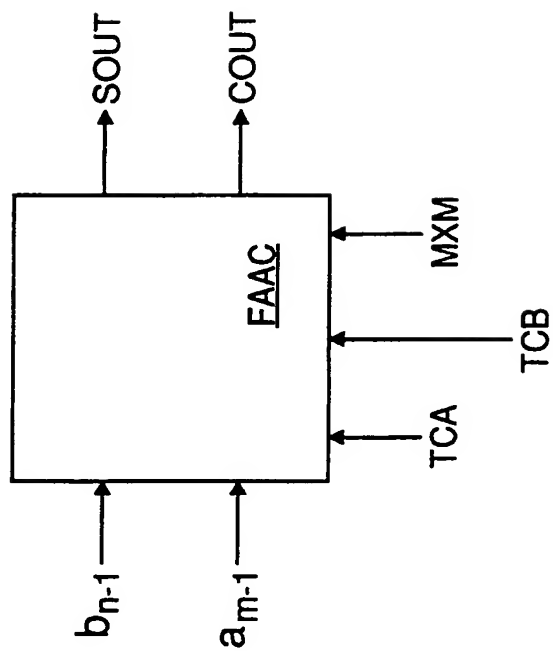


FIG. 19b



31/36

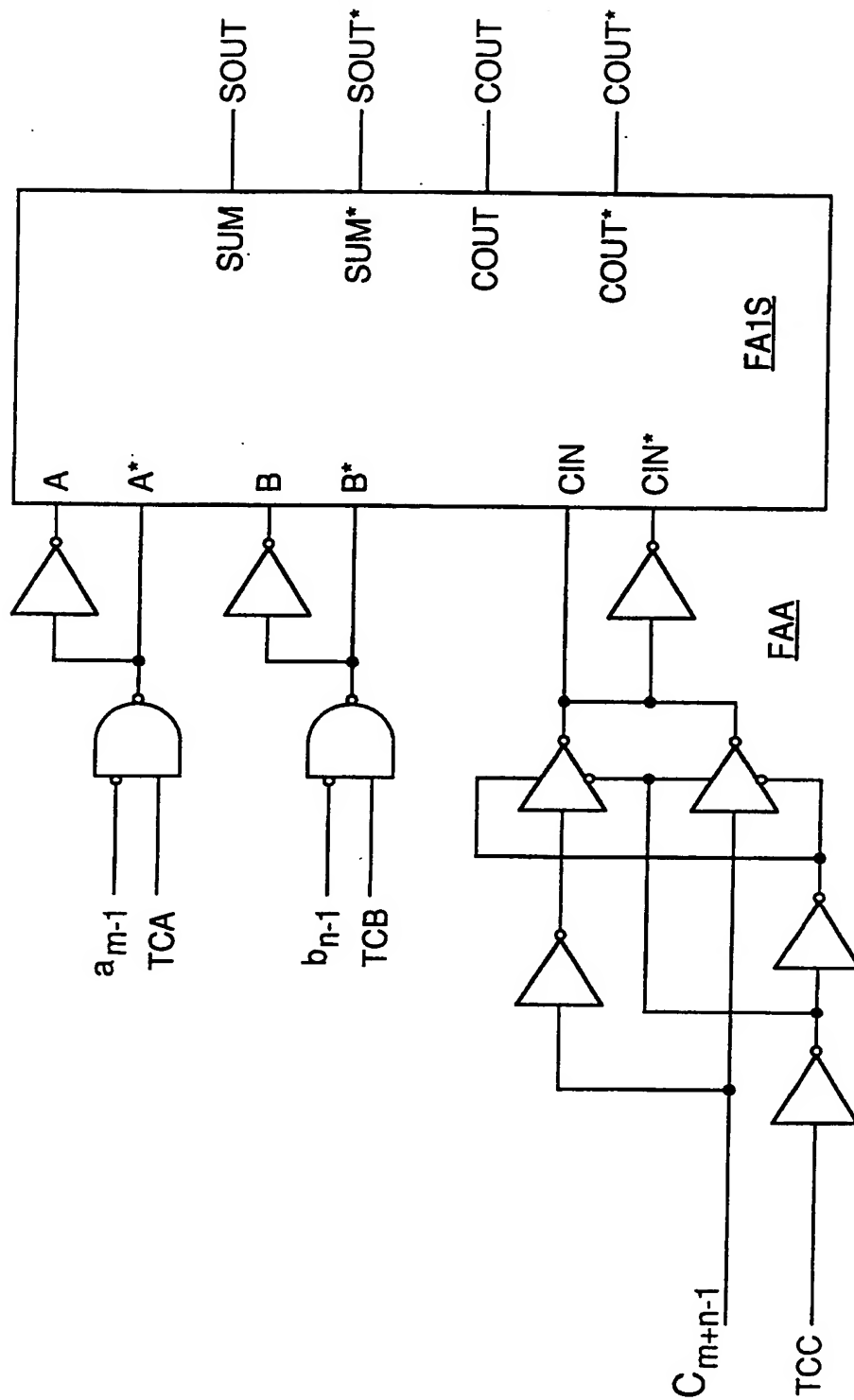


FIG. 20a

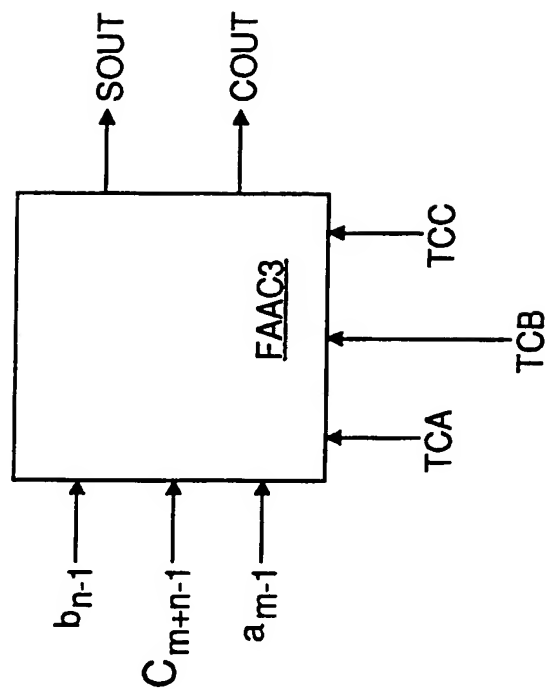


FIG. 20b

FIG. 21a

33/36

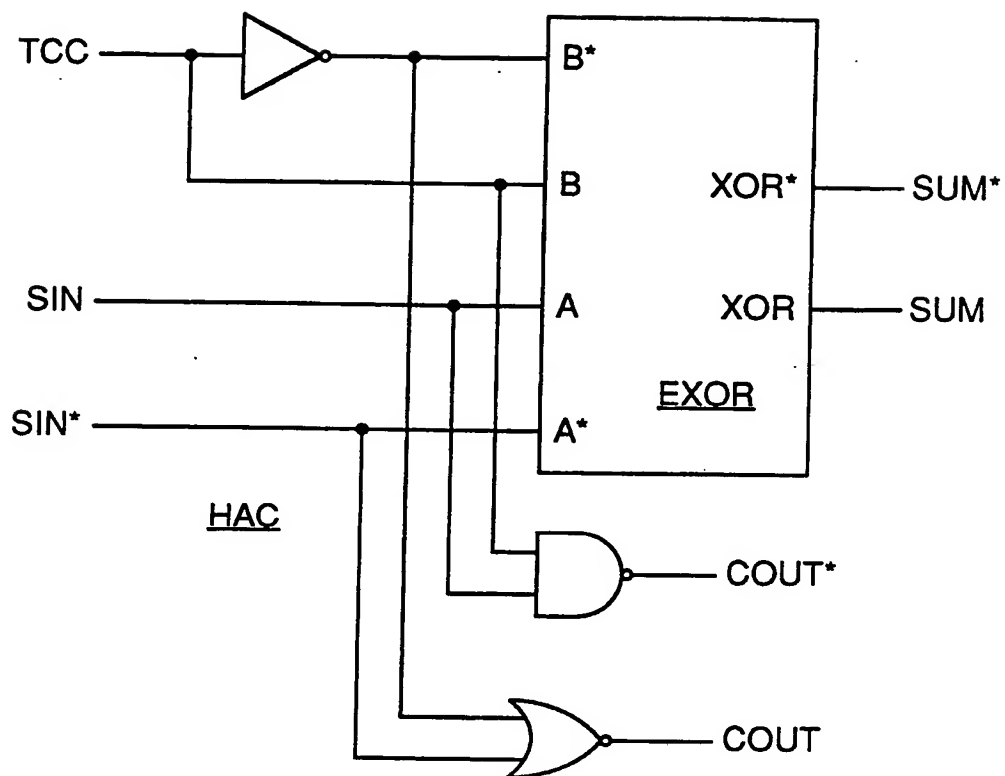
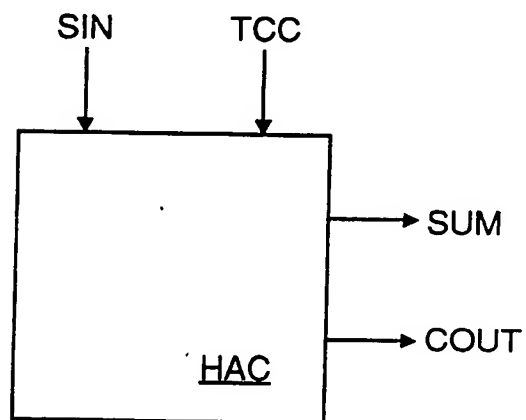


FIG. 21b



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FIG. 22a

34/36

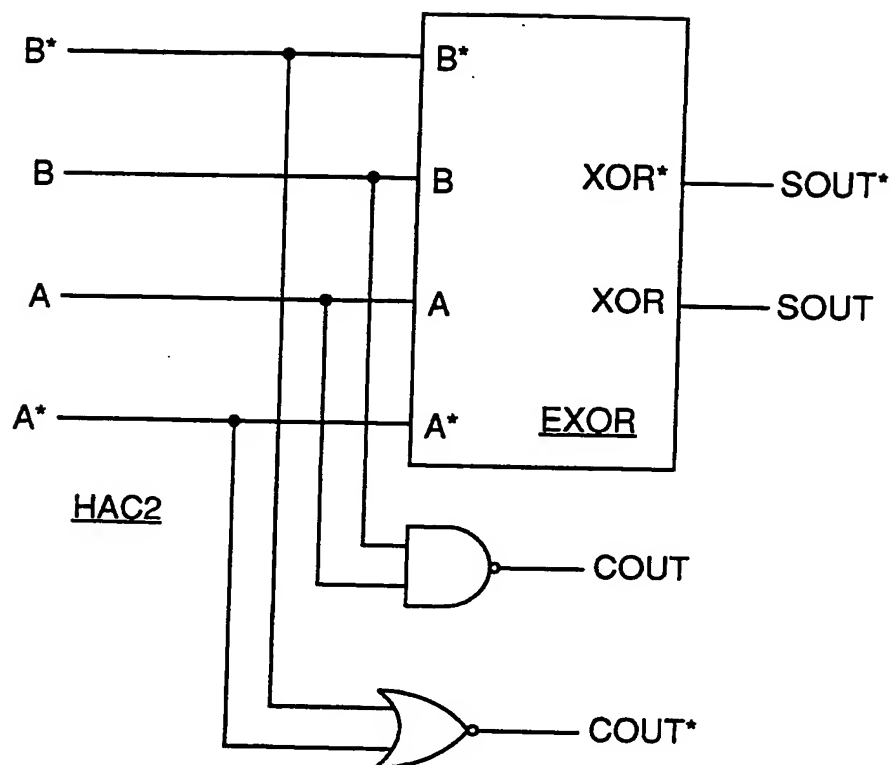


FIG. 22b

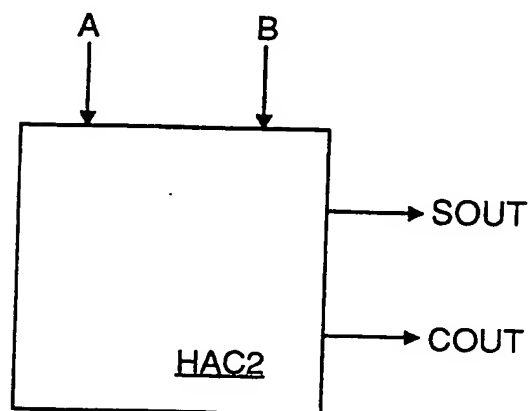


FIG. 23

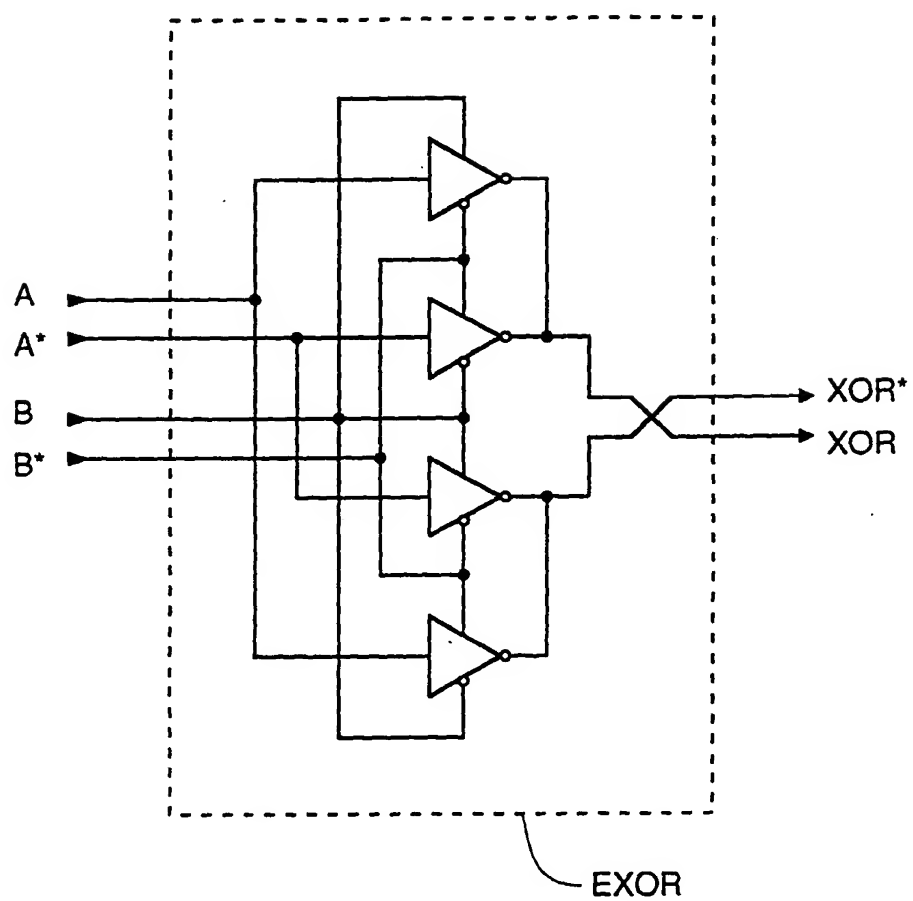
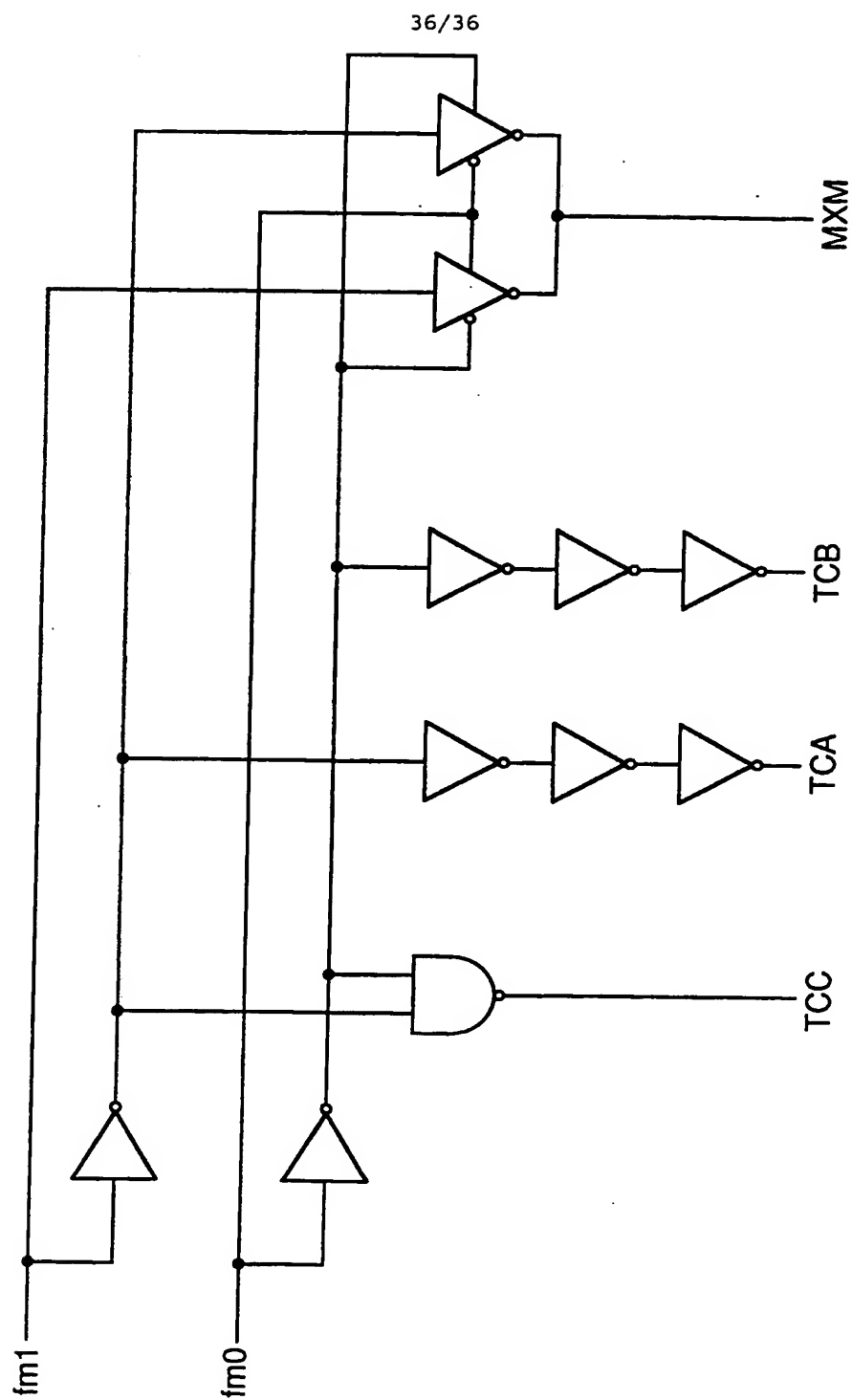


FIG. 24



INTERNATIONAL SEARCH REPORT

International Application No.

PCT/US91/00823

I. CLASSIFICATION OF SUBJECT MATTER (if several classification symbols apply, indicate all) *

According to International Patent Classification (IPC) or to both National Classification and IPC

IPC(5): G06F 7/52

US CL.: 364/757,758

II. FIELDS SEARCHED

Minimum Documentation Searched *

Classification System

Classification Symbols

US 364/757,758,759

Documentation Searched other than Minimum Documentation
to the Extent that such Documents are Included in the Fields Searched *

III. DOCUMENTS CONSIDERED TO BE RELEVANT ¹⁴

Category * Citation of Document, ¹⁵ with indication, where appropriate, of the relevant passages ¹⁷ Relevant to Claim No. ¹⁶

A	US, A, 4,706,210 (SNELLING et al.) 10 November 1987 See entire document.	1-29
A	US, A, 4,831,577 (WEI et al.) 16 May 1989 See entire document.	1-29
A	US, A, 4,839,848 (PETERSON et al.) 13 June 1989 See entire document.	1-29
A	US, A, 3,866,030 (BAUGH et al.) 11 February 1975 See entire document.	1-29

* Special categories of cited documents: ¹³

"A" document defining the general state of the art which is not considered to be of particular relevance

"E" earlier document but published on or after the international filing date

"L" document which may throw doubt on priority claim(s) or which is cited to establish the publication date of another citation or other special reason (as specified)

"O" document referring to an oral disclosure, use, exhibition or other means

"P" document published prior to the international filing date but later than the priority date claimed

"T" later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention

"X" document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step

"Y" document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art.

"A" document member of the same patent family

IV. CERTIFICATION

Date of the Actual Completion of the International Search *

10 APRIL 1991

International Searching Authority ¹

ISA/US

Date of Mailing of this International Search Report *

22 MAY 1991

Signature of Authorized Officer ¹⁸

D.H. MALZAHN